



University of Bielsko- Biała

Faculty of Mechanical Engineering and Computer Science

*Thermo-energetical Master Thesis*

# Two-dimensional modeling of steady state heat transfer in solids with use of spreadsheet (MS EXCEL)

Accuracy and effectiveness study of the method in  
application involving a finned surfaces

**Luis García Blanch**

**Tutor: Professor Andrzej Sucheta, Ph.D., D.Sc.**

*Spring 2011- Bielsko-Biała, Poland*



## BRIEF SUMMARY

In order to resolve both temperature distribution and heat flux in bodies whose geometries are not simple the analytical solutions are limited. In unsteady conditions are needed methods that would allow to calculate temperatures distribution within a three-dimensional heat conducting body of any shape.

In this project, the study is focused on two-dimensional modeling steady state. The target is using the MS EXCEL program specifying iterative calculations in order to get a temperature distribution of a concrete shape of piece.

The aim of the thesis is offering to the universities an alternative way of calculating heat transfer by numerical method. Nowadays programs like ANSYS, which costs use to be quite expensive, provide the students of a quick and accurate sort of solving this kind of problems, however, in the work done below is shown how MS EXCEL is successful in the same type of calculations. The advantage of the usage of this program is clear, most of the universities can afford provide their students MS EXCEL.

The accomplishment of one of the tests done is compared with the results given by ANSYS, with satisfactory conclusions.

## CHRONOLOGY AND ACKNOWLEDGEMENTS

The accomplishment of this thesis has been possible thanks to the agreement between the universities ETSEIB (Technical Superior School of Industrial Engineering of Barcelona) and ATH (Akademia Techniczno-Humanistyczna, Bielsko Biala).

I have coursed the biggest part of my studies in Barcelona (first four courses and a half), but I have dedicated my degree's last five months in the performance of this thesis at the university of Bielsko Biala under a ERASMUS exchange program during the spring of 2011.

After choosing the *Thermo-energetical* specialization and once the ERASMUS program was available I had the chance of accomplish my final thesis in the Faculty of Mechanical Engineering and Computer Science of ATH. This thesis has been entirely carried out with the help of the teachers and doctors of the department as well as with the department tools needed during the five months process.

I wish to thank my tutor, Professor Andrzej Sucheta from Department of Internal Combustion Engines and Vehicles, for all the attention received during the thesis performance as well as all the information and knowledge provided in each step carried out in the thesis. As a teacher, Professor Andrzej Sucheta has helped me in the learning and remembering of some essential concepts needed in the making and understanding of the project. In addition, I would like to thank Professor Andrzej Sucheta for the suggestion of the subject and the idea of developing such a thesis in the required time given by the ERASMUS exchange program. Also I would like to remark that Professor Sucheta has been available and disposed to discuss the thesis parts every week during my stay in Poland and the team work has always been easy with him.

I wish to thank Dr. Krzysztof Sikora, Ph.D for the indications given focused in the application of the method described in this thesis with MS EXCEL. Also, Dr. Sikora has explained me how ANSYS works for the performance carried out in the chapter 5 of this project. Dr. Sikora has designed the piece in ANSYS and the comparison between the programs has been possible thanks to the Dr. Sikora work. Dr. Sikora has designed as well the macro used to run the iterative calculations in MS EXCEL.

Eventually, I would like to thank all the staff of International Relations in ATH for making possible this experience and helping me in all the required issues to fulfill with my intention of developing my final thesis in Poland.



<b>1</b>	<b>COMPARISON: ANALYTICAL AND NUMERICAL MODEL .....</b>	<b>1-9</b>
1.1	FOURIER-KIRCHHOFF EQUATION.....	1-9
1.2	ANALYTICAL MODEL.....	1-12
1.3	NUMERICAL METHODS .....	1-14
<b>2</b>	<b>DESCRIPTION OF THE METHOD .....</b>	<b>2-16</b>
2.1	INTRODUCTION.....	2-16
2.2	HOW TO PERFORM NUMERICAL METHOD USING MS EXCEL. ....	2-19
2.2.1	<i>Spreadsheets</i> .....	2-19
2.2.2	<i>Formulas</i> .....	2-20
2.2.3	<i>Format</i> .....	2-21
2.3	CONDUCTANCE.....	2-23
2.4	TEMPERATURE AND ITERATIVE CALCULS .....	2-24
2.4.1	<i>Usage of Macros</i> .....	2-25
2.5	PREDICTING AND CONVERGING.....	2-27
2.6	TEMPERATURE DISTRIBUTION GRAPHIC.....	2-29
<b>3</b>	<b>EXPERIMENTAL CASES.....</b>	<b>3-31</b>
3.1	MODELING .....	3-31
3.2	FIRST AND THIRD KIND OF BOUNDARY CONDITION .....	3-33
3.2.1	<i>Forced Convection (<math>\alpha = 100</math>)</i> .....	3-35
3.2.2	<i>Natural Convection</i> .....	3-41
3.2.3	<i>Comparison between forced and natural convection</i> .....	3-46
3.3	HEAT FLUX FROM THE BASE AND ISOLATION. SECOND KIND OF BOUNDARY CONDITION. ....	3-47
3.3.1	<i>Heat: Isolation &amp; Generation</i> .....	3-48
3.3.2	<i>Results</i> .....	3-49
<b>4</b>	<b>HEAT TRANSFER FROM FINNED SURFACES.....</b>	<b>4-51</b>
4.1	THEORY INTRODUCTION .....	4-51
4.2	EFFICIENCY & EFFECTIVENESS OF THE FINS.....	4-55

4.2.1	<i>Fin efficiency</i> .....	4-55
4.2.2	<i>Fin effectiveness</i> .....	4-58
4.2.3	<i>Overall fin effectiveness</i> .....	4-59
4.2.4	<i>Results and comparison</i> .....	4-61
<b>5</b>	<b>ANALYSIS WITH ANSYS</b> .....	<b>5-63</b>
5.1	DESCRIPTION OF THE METHOD.....	5-63
5.1.1	<i>Geometry and meshing</i> .....	5-64
5.1.2	<i>Setup</i> .....	5-67
5.2	RESULTS.....	5-70
<b>6</b>	<b>CONCLUSIONS</b> .....	<b>6-71</b>
6.1	COMPARISON BETWEEN ANSYS AND MS EXCEL .....	6-71
6.2	EFFECTIVENESS AND ACCURACY IN MS EXCEL .....	6-73

## Symbols

### Latin Symbols

Symbol	Units	Description
$A$	[ m <sup>2</sup> ]	area
$a$	[ m <sup>2</sup> /s ]	thermal diffusivity
$c_p$	[ J/(kg·K) ]	specific heat at constant pressure
$E_u$	[J]	amount of internal energy
$g$	[ m/s <sup>2</sup> ]	gravitational acceleration
$k$	[ W/m <sup>2</sup> K ]	thermal conductance
$k_D$	[ W/m <sup>2</sup> K ]	thermal down conductance
$k_L$	[ W/m <sup>2</sup> K ]	thermal left conductance
$Q$	[W]	predicted value of rate energy transfer (Tabs. 2,3,4,5,6,7,9)
$\dot{Q}$	[W]	rate of energy transferred
$\dot{q}_A$	[ W/m <sup>2</sup> ]	rate of energy generation per unit area
$\dot{Q}_\infty$	[W]	predicted value of rate energy transfer
$\dot{q}_v$	[ W/m <sup>3</sup> ]	rate of energy generation per unit volume
$\dot{Q}_x$	[W]	rate of energy transfer in x direction
$R$	[K/W]	thermal resistance
$r$	[ - ]	common ratio
$r$	[m]	radius
$R_{t,cond}$	[K/W]	thermal conduction resistance
$T$	[K]	temperature
$t$	[s]	time
$T_f$	[K]	film temperature
$T_\infty$	[K]	free stream temperature
$T_s$	[K]	surface temperature
$V$	[m <sup>3</sup> ]	volume
$p$	[m]	perimeter
$A_c$	[m <sup>2</sup> ]	cross sectional area
$T_b$	[K]	base temperature



### Greek Symbols

$\alpha$	[ W/(m <sup>2</sup> ·K) ]	convection heat transfer coefficient
$\beta$	[K <sup>-1</sup> ]	inverse of film temperature
$\delta$	[mm]	thickness
$\varepsilon$	[ - ]	effectiveness
$\eta$	[ - ]	efficiency
$\vartheta$	[K]	temperature difference
$\lambda$	[ W/(m·K) ]	thermal conductivity
$\nu$	[m <sup>2</sup> /s]	kinematic viscosity
$\rho$	[ kg/m <sup>3</sup> ]	density

### Dimensionless numbers

Gr	Grashof number
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number

# 1 Comparison: Analytical and Numerical Model

## 1.1 Fourier-Kirchhoff Equation

The relation between the heat energy, expressed by the heat flux  $\vec{q}$ , and its intensity, expressed by temperature  $T$ , is the essence of the Fourier Law, the general character which is the basis for analysis of various phenomena of heat considerations. The analysis is performed by the usage of the heat conduction equation of Fourier-Kirchhoff. To derive this equation it is considered the process of heat flow by conduction from a solid body of any shape and volume  $V$  located in an environment of temperature  $T_o(t)$  [1].

Two processes can take place: the generation of heat inside the body and the heat transfer between the body and its environment. Therefore, the total amount of heat  $dQ$  in a specific  $dV$  (Volume differential) corresponds to the sum of both terms:

Conduction: 
$$dQ_1 = -\lambda \cdot \frac{dT}{dn} \cdot d\vec{A} = -\vec{q} \cdot d\vec{A} \quad (1.1)$$

Generation: 
$$dQ_2 = \dot{q}_v \cdot dV \quad (1.2)$$

The total amount of  $dQ$  must be equal to the internal energy change. From the First Law of the Thermodynamics:

The expression of  $dE_u$  is known:

$$dE_u = dQ_1 + dQ_2 \quad (1.3)$$

$$dE_u = \rho \cdot dV \cdot c_p \cdot \frac{\partial T}{\partial t} \quad (1.4)$$

If the equations join the same expression and are developed integrating in the whole volume and crossing area, we find:

$$\int_V \rho \cdot c_p \cdot \frac{\partial T}{\partial t} \cdot dV = \int_A -\vec{q} \cdot d\vec{A} + \int_V \dot{q}_v \cdot dV \quad (1.5)$$

On application of the Gauss-Ostrogradsky theorem, which states that the surface integral of a vector is equal to the volume integral of the divergence of the vector, we can write:

$$\int_A -\vec{q} \cdot d\vec{A} = \int_V \nabla(-q) \cdot dV \quad (1.6)$$

We develop the whole expressions with the proper replacements:

$$\int_V \left[ \rho \cdot c_p \cdot \frac{\partial T}{\partial t} - \nabla(\lambda \cdot \nabla T) - \dot{q}_v \right] dV = 0 \quad (1.7)$$

$$\left[ \rho \cdot c_p \cdot \frac{\partial T}{\partial t} - \nabla(\lambda \cdot \nabla T) - \dot{q}_v \right] = 0 \quad (1.8)$$

Eventually:

$$\frac{1}{a} \frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}_v}{\lambda} \quad (1.9)$$

where

$$a = \frac{\lambda}{c_p \cdot \rho} \text{ is the thermal diffusivity}$$

In the next chapter is proposed one way of solving the equation (1.9) using energy balance Numerical Methods.

### **Boundary Conditions**

The solution of the temperature distribution depends on the physical conditions existing at the boundary of the medium [2].

In the next paragraphs are explained the typical three classified boundary conditions for a one dimensional system for simplicity.

#### ***- Prescribed Temperature at the Boundary***

In this situation the temperature at the body surface  $T_s$  is known for any instant. The condition is called **Dirichlet Condition** or boundary condition of the **first kind**. Most of the real situations related with this condition are based in an intensify heat transfer between the body and the surrounding. For instance, a body surrounded by a melting

solid or a boiling liquid. In both examples the temperature is constant at the boundary and one may assume that the temperature remains the same in the surface of the piece. We may brief the condition with the next expressions:

$$T(x = 0, t) = T_s \quad (1.10)$$

where  $T_s$  represents the (given) surface temperature.

We assume that the heat transfer is intensive when the convective coefficient  $\alpha$  is moderately high but the thermal conductivity is low and the heat conducting body is large. In these conditions we also assume  $T_\infty$  and the temperature of the surface constant.

- *Prescribed Heat Flux at the Boundary*

In this case is given a prescribed heat flux  $\dot{q}_s$  at the surface. Using Fourier's law we can define  $\dot{q}_s$  as:

$$+\lambda \cdot \frac{\partial T(x, t)}{\partial x} \Big|_{x=0} = \dot{q}_s(t) \quad (1.11)$$

The given heat flux boundary conditions is called **Neumann condition**, or boundary condition of the **second kind**. Furthermore, there is a specific case of this condition and corresponds to the adiabatic surface. In this situation:

$$\frac{\partial T(x, t)}{\partial x} \Big|_{x=0} = 0 \quad (1.12)$$

- *Convective Exchange at the Boundary*

The boundary conditions of the **third kind** correspond to the existence of convective heating or cooling. The heat dissipated or won by convection is the same that fluxes across the body in one specific point:

$$\lambda \cdot \frac{\partial T(x, t)}{\partial n} \Big|_s = \alpha \cdot (T_s - T_\infty) \quad (1.13)$$

The **third kind** of boundary condition is also called **Newton** or **Robin Condition**.

## 1.2 Analytical Model

The starting point is focused in the heat diffusion equation. From this equation the objective is solve and analyze the heat flux distribution, the temperature distribution and the temperature gradient of bodies of different simple shapes.

Fourier law provides the first step from where the rest of considerations are determined:

$$\dot{Q}_x = -\lambda \cdot A \cdot \frac{dT}{dx} \quad (1.14)$$

The equation describes the heat transfer rates. The heat is transferred in the direction of decreasing temperature and cross the Area perpendicularly. The thermal conductivity  $\lambda$  is a transport property of the material since it provides an indication of the rate at which energy is transported by the diffusion process. If the Fourier's law is considering each direction of the space, is possible to write it in a vector form:

$$\vec{q} = -\lambda \cdot \left( \frac{\partial T}{\partial x} \cdot \vec{i} + \frac{\partial T}{\partial y} \cdot \vec{j} + \frac{\partial T}{\partial z} \cdot \vec{k} \right) \quad (1.15)$$

The expression relates the heat flux across a surface to the temperature gradient in a direction perpendicular to the surface.

Analytical expressions describe both temperature distribution and heat transfer rate in bodies conducting heat, in this thesis, focused in steady-state conditions. The expressions are classified depending on the shape of the body that is required to analyze.

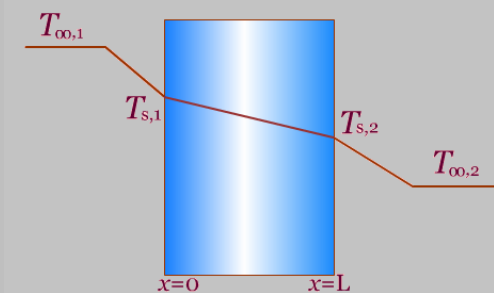
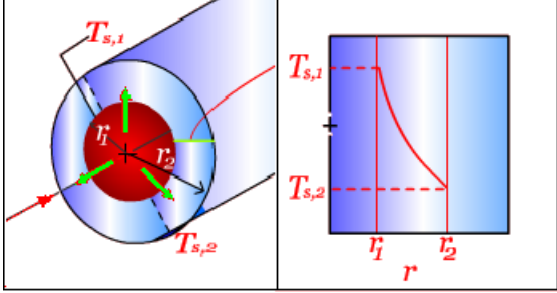
From the Fourier's law until the concept of *Thermal Resistance* based in the comparison between heat flux and Current, through different considerations such as consider the conductivity of the materials independent of the direction the heat flows, it is possible to define the next general expression in order to calculate heat flux:

$$\dot{Q} = \frac{\Delta T}{\Sigma R} \quad (1.16)$$

$\Sigma R$ . means sum of both convection and conduction resistances in the heat flux direction.

In order to summarize as much as possible is presented below a table with the conduction resistances expressions given for the most important shapes. For steady-state heat conduction with no energy generation within the wall

Table 1. Heat transfer equation and resistance expression in plane and cylindrical walls

Shape	Heat transfer equation	Resistance expression
<p><b>Plane Wall</b></p> 	$\frac{d}{dx} \left( \lambda \cdot \frac{dT}{dx} \right) = 0$	$R_{t,cond} = \frac{L}{\lambda \cdot Area}$
<p><b>Cylindrical Wall</b></p> 	$\frac{d}{dr} \left( \lambda \cdot r \cdot \frac{dT}{dr} \right) = 0$	$R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi \cdot \lambda \cdot L}$

The equations are applicable for bodies with such as simple geometries. Also it is possible to arrive to analytical solutions describing temperatures in two or three-dimensional bodies, either for steady or unsteady conditions, however, the requirement does not change: easy geometries. Consequently, the studies are very restricted and it is needed to develop a method able to solve any shape possibility with accuracy.

As it is been remarked, the analytical solutions are effective for three-dimensional bodies as well, the methodology used for the determination of the proper equations are based in the *infinite series*. As a result, the expressions get more difficult and, like in the previous ones, it is necessary to use known boundary conditions in order to obtain the final solution

Eventually, the conclusion is that every analytical solution comes from the phenomenological Fourier's Law and is specified following mathematical required steps. Furthermore, the most remarkable point of this chapter is the impossibility to apply analytical solutions to this project, due to the complexity of the piece shape and the required accuracy in every result wanted to achieve.

Engineers should have a tool that would allow for calculating temperature distribution even in a three-dimensional heat conducting body of an arbitrarily specified shape [2].

## 1.3 Numerical Methods

Numerical Method is based in the computer calculations after specifying the machine the proper boundary conditions of the piece wanted to study.

Heat conduction is governed by the heat conduction equation that in Cartesian coordinated takes the form:

$$\frac{\partial}{\partial x} \left( \lambda \cdot \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \cdot \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \cdot \frac{\partial T}{\partial z} \right) + q_v = \rho \cdot c_p \cdot \frac{\partial T}{\partial t} \quad (1.17)$$

This general equation contains the three basic terms: diffusion term, the source term and the unsteady term.

The importance of the expression is the allowance to provide the value of temperature at a finite number of locations (grid points) in the calculation domain. As it will be explained in the next chapters, the domain of the studied piece is divided in grid points with a specific length, represented by the cells of the spreadsheet in MS EXCEL. Consequently, the firsts steps that must be followed to present a Numerical Method study are discretize (divide in grid points) correctly the body.

Experienced computer programs like ANSYS, offer several options to discretize any piece, however, MS EXCEL requires to set manually the disposition of the cells. In order to fit MS EXCEL limitation, the discretized body shows a Cartesian, uniform grid. In spite of this fact,

it is important to comment that discretization using triangular shapes would be possible in other programs and the solution would have to be equal.

From the equation (1.17) are derived the discretization equations that will join the grid points in which the piece will be divided. A discretization equation is an algebraic relation connecting the temperature varies between the grid points with temperatures at the neighbor grid points.

The *algebraic connection* can be carried out in several ways. This thesis purposes a methodology that will be explained in the next chapters. It will be built a simple numerical method for solving the set of algebraic equation (1.17), modified because of the disappearance of the unsteady term, due to the aim of the project study. Thus, the equation to resolve is:

$$\frac{\partial}{\partial x} \left( \lambda \cdot \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \cdot \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \cdot \frac{\partial T}{\partial z} \right) + q_v = 0 \quad (1.18)$$

As an introduction for the next steps in the project, it is necessary to remark the unavoidable need of using iterative calculation. If the aim of this method is provide a solution using every temperature belonging to each grid point, it will be necessary to update the temperature results with the intention of using it afterwards [2]

## References

- [1] Wojciech Zielenkiewicz, Eugeniusz Margas, "*Theory of calorimetry*", 2002 Kluwer Academic Publishers
- [2] Roman Weber, "*Lectures Notes in Heat Transfer, 2<sup>nd</sup> Edition*", 2004 Clausthal-Zellerfeld



## 2 Description of the method

### 2.1 Introduction

In a general view the method consists in the use of the cells of the spreadsheet in MS EXCEL as a way of discretization for a given piece. Each cell represents a control volume with a specified length in the two directions of the sheet ( $\Delta x$ ,  $\Delta y$ ). Logically, the results are more accurate as the number of cells used is bigger. The usage of these cells provides a simple way to write the needed equations for both temperature and heat flux.

In the next paragraphs is presented a specific way to solve by Numerical Method the equation (1.9) explained in chapter 1.1. The cases used to carry out with the method are considered steady in time, thus, the equation becomes:

$$\nabla^2 T + \frac{\dot{q}_v}{\lambda} = 0 \quad (2.1)$$

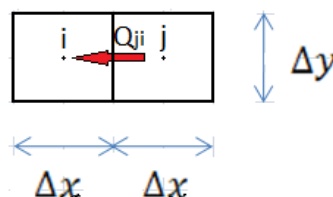
Moreover, is necessary to find the expressions that will represent the algebraic connection between the defined grid points for each case of heat transfer (conduction, convection, etc.).

Furthermore, the description of the cells by a given value of  $\Delta x$  and  $\Delta y$  is needed in every sort of Numerical Method applied. The pieces studied in this project are considered to be 2D; therefore, the depth of the body in each step will be equal to 1 ( $\Delta z = 1$ ). However, it will be indicated in the equations in order to write them down properly.

#### Energy balance equation

If we consider two squared cells together is possible to write the conduction equation that will take place between them evaluating the resistance that the heat flux will come across between two different temperatures:

Figure 1. Heat transfer between cells



$$\dot{Q}_{ji} = \frac{T_j - T_i}{R_{js} + R_{si}} = k_{ji}(T_j - T_i) \quad (2.2)$$

where:

$R_{js}$  is the resistance from the center of the grid point  $j$  to the border between grid points ( $S$ ).

$R_{si}$  is the resistance from the border  $S$  to the center of the grid point  $i$ .

$T_j$  is the temperature of the grid point  $j$  and  $T_i$  the temperature of the grid point  $i$ .

$k_{ji}$  is the thermal conductance between nodes or grid points.

The expression of each resistance in this case follows the equation for *plane walls* saw in chapter 1.1:

$$R_{js} = \frac{\Delta x}{2 \cdot \lambda_j \cdot \Delta y \cdot \Delta x} \quad R_{is} = \frac{\Delta x}{2 \cdot \lambda_i \cdot \Delta y \cdot \Delta x} \quad (2.3)$$

where:

$\Delta x$  is the width of every node

$\Delta y$  is the height of every node

$\lambda_i$  is the thermal conductivity coefficient in cell  $i$

$\lambda_j$  is the thermal conductivity coefficient in cell  $j$

Serial resistances add up and it is possible to express it with the next dependence using thermal conductivity:

$$\frac{1}{k_{ji}} = \frac{\Delta x}{\Delta y} \left( \frac{1}{2\lambda_j} + \frac{1}{2\lambda_i} \right) \quad (2.4)$$

Each cell in the internal points is surrounded by another 4 cells, so that we can express the energy balance taking into account the action of heat sources in the whole volume of the cells as:

$$\sum_j \dot{Q}_{ji} + \dot{q}_{Vi} \cdot \Delta x \cdot \Delta y = 0 \quad (2.5)$$

The temperature of a cell "i" may be finding out from the previous expression as:

$$T_i = \frac{\sum_{j=1}^4 k_{ji} T_j + \dot{q}_{Vi} \Delta x \Delta y}{\sum_{j=1}^4 k_{ji}} \quad (2.6)$$

In the border cells the heat equation will depend on the boundary conditions given in the surrounding. The heat conducted between inside cells was expressed as  $\dot{Q}_{ji}$ , in the next expressions the heat exchange between the surrounding and any cell  $i$  is expressed as  $\dot{Q}_{Bi}$ .

- 1<sup>st</sup> kind of Boundary Condition. Prescribed temperature.

$$\dot{Q}_{Bi} = \frac{T_B - T_i}{R_{Bi}} = \frac{T_B - T_i}{\frac{\Delta x}{2\lambda_i \cdot \Delta y \cdot 1}} = k_{Bi} \cdot (T_B - T_i) \quad (2.7)$$

$$\text{where} \quad \frac{1}{k_{Bi}} = \frac{\Delta x}{2\lambda_i \Delta y} \quad (2.8)$$

- 2<sup>nd</sup> Kind of Boundary Condition. Heat flux  $\dot{q}$ .

$$\dot{Q}_{Bi} = \dot{q} \cdot \Delta y \cdot 1 \quad (2.9)$$

- 3<sup>rd</sup> Kind of Boundary Condition. Convection.

In this case we find two kinds of resistances between the central point of any boundary cell and the environment. The first one corresponds to heat fluxing by conduction and the second one by convection:

$$\dot{Q}_{Bi} = \frac{T_0 - T_i}{R_{oB} + R_{Bi}} = k_{oi} \cdot (T_0 - T_i) \quad (2.10)$$

$$\text{where} \quad \frac{1}{k_{oi}} = \frac{1}{\alpha \cdot \Delta y \cdot 1} + \frac{\Delta x}{2\lambda_i \Delta y \cdot 1} \quad (2.11)$$

$T_0$  is the temperature of the environment

$R_{oB}$  is the convection resistance between the boundary and the environment

$R_{Bi}$  is the conduction resistance between the central point of the cell and its boundary.

## 2.2 How to perform Numerical Method using MS EXCEL.

In the chapter number 3 we will find out the temperature distribution and the heat dissipated by a specific piece. MS EXCEL provides a methodology to achieve these results. In the next paragraphs it is presented the main steps the user must follow to reach the wanted solution. Moreover, the facilities that the MS EXCEL offer will be explained as well as the inconvenients of using a program which characteristics are not focused in heat transfer.

### 2.2.1 Spreadsheets

One of the main steps the user should follow to achieve a good performance of an MS EXCEL heat transfer solving is organize the information and the data which will fill the document in order to carry out with the proper calculations.

It will be necessary to create in MS EXCEL different sheets depending on the given data for solving the problem, the temperature iterative calculation, the heat transfer and the results.

Every spreadsheet, but the ones concerning to the results, should have the geometry of the piece drawn in the same cells in order to be coherent with the formulas and its calculations.

#### Organization

The properties of the material such as thermal conductivity will be presented in a spreadsheet called "k". From there, every single data concerning to the characteristics of the material or the conditions of the environment such as convection coefficient used may be called even from different spreadsheets. In this spreadsheet not only the piece will be drawn but also the tables with the whole information of the problem given: coefficients, dimensions...

The temperature spreadsheet (T) will contain formulas for T calculations and the current values of temperature throughout the piece, updated after iterations. Also will be filled with the temperatures in the surrounding which define any boundary condition

The conductance spreadsheets ("D", "G", "P" and "L") will show through the piece the values of the conductance calculated in directions arbitrary: down, up, right and left from the cell considered

Spreadsheet “q” includes the heat transfer values between the border cells and the surrounding and also the values of the volume heat source. The rest of the piece in this spreadsheet has the value of 0. To pool cells border with Second Kind of Boundary Condition we have to plus heat from source areas. For example: when density of source volume comes of Second Kind of Boundary Condition is possible to bring them the value of:

$$q_i = \dot{q}_{Vi}(\Delta x \Delta y \cdot 1) + \dot{q}_{Ai}(\Delta x \cdot 1) \quad (2.12)$$

The results may be presented separated in three spreadsheets: one dedicated to the temperature distribution graphic, other with the predicting table of heat values and the last one, in this case, filled with the efficiency and effectiveness calculations.

## 2.2.2 Formulas

The expressions found in chapter 2.1 may be easily written down in MS EXCEL. Once the geometry of the piece is modeled by the usage of the cells needed, the expressions of conductivity, conductance, heat transfer and temperature must be specified in each cell. That involves the “copy” of different equations in lots of cells with few changes related with the position of the cells. MS EXCEL provides during the operation of “copy-paste” the changing address automatically. If there is some parameter that is not wanted to change the address is possible to use the command \$. For instance, the copy of the value of conductivity constant in every cell of the whole piece:

Figure 2. Conductivity throughout the cell

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
3	Afin	0,4755 m2																			
4	Δx=Δy	0,00025 m																			
5	λ	200 W/mK																			
6	α	100 W/m2K																			
7	Ltot	0,4905 m																			
8	Atot	0,4905 m2																			
9	Aunfin	0,015 m2																			
10	Anofin	0,0036429 useless																			
11																					
12	Δi	Delta.i	Qi(totnofin)																		
13	0,008	0,006	31,861743																		
14	0,0025	0,007	9,9567946																		
15	0,0025	0,0085	9,9567946																		
16	0,0025	0,00925	9,9567946																		
17	0,0025	0,00975	9,9567946																		
18	0,0025	0,01	9,9567946																		
19	0,005	0,01025	19,913589																		
20																					
21																					

The operation of copy formulas has been carried out with the option “paste as”. In this option a window opens and is possible to paste exclusively the formula.

In this way is possible to write in one side of the screen the characteristics of the material such as thermal conductivity,  $\Delta x, \Delta y$  and even the convection coefficient of the environment. Every time the user needs it, is possible to appeal to the specific cells. In our case we have written in the left superior corner as is shown in the previous image.

Furthermore, is totally available the chance of using different spreadsheets during the operation of copy. In this case, the MS EXCEL will indicate in the expression the spreadsheet where the value comes from. In the next image is shown a little example:

,Figure 3. Conductance

E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
200	200	200	200								200	200	200	200					
200	200	200	200								200	200	200	200					
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400

The current location of the spreadsheet is the one named “D” and the formula shown in the space for functions ( $f_x$ ) makes reference to the spreadsheet named as “k” by the indication of “k!” before the cell used.

That is why is important to copy the geometry modeled using always the same cells in every spreadsheet. This operation may be carry out by a manual copying or copying directly the whole spreadsheet into other with a different name.

Most of the formulas needed may be modeled easily by this way; however, temperature expression will be different due to the dependence between the cells. In the chapter 2.4 is explained how MS EXCEL makes possible the iterative calculation.

### 2.2.3 Format

The pool of the piece must be clearly distinguished from the border, thus, after drawing the geometry using border options in MS EXCEL, is very useful to specify each kind of boundary condition by using not only the temperatures or the expressions needed but also a proper format.

For instance, when the user wants to specify the First Kind of Boundary Condition is possible to dispose in the temperature spreadsheet of the prescribed temperature in all the borders (out of the piece pool) where the condition may be found. Also will be important because the needed formulas will require the value of these temperatures.

In the third case of boundary condition (convection) should be indicated not only the temperature in its spreadsheet but also the convection coefficient of the environment. It may be added in the conductivity spreadsheet, where all the useful data to solve the thermal problem is.

The cells inside the piece pool contain the equation for the calculation of temperature (in the case of its spreadsheet) or the values of conductivity and conductance, as well as the heat source already mentioned.

The images below shows the disposal of the temperatures and coefficient mentioned. The used format is also important to distinguish between different temperatures. In addition, is useful to notice where the piece pool ends.

Figure 4. Temperature Spreadsheet

	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
124		20	59,094297	59	59	59	59	59	59	59	59	59	59	59	59	59
125		20	59,188381	59	59	59	59	59	59	59	59	59	59	59	59	59
126		20	59,282287	59	59	59	59	59	59	59	59	59	59	59	59	59
127		20	59,376159	59	59	59	59	59	59	59	59	59	59	59	59	59
128		20	59,470144	59	59	59	59	59	59	59	59	60	60	60	60	60
129		20	59,564401	60	60	60	60	60	60	60	60	60	60	60	60	60
130		20	59,659119	60	60	60	60	60	60	60	60	60	60	60	60	60
131		20	59,754543	60	60	60	60	60	60	60	60	60	60	60	60	60
132		20	59,851049	60	60	60	60	60	60	60	60	60	60	60	60	60
133		20	59,949297	60	60	60	60	60	60	60	60	60	60	60	60	60
134				60	60	60	60	60	60	60	60	60	60	60	60	60
135																
136																
137																
138																

Figure 5. conductivity Spreadsheet

$\Delta x = \Delta y$	0,00025 m
$\lambda$	200 W/mK
$\alpha$	100 W/m2K

## 2.3 Conductance

The conductance in the inside pool points (those which does not share border) are calculated as a harmonic average of  $k$ 's using this mathematical option in MS EXCEL. We dispose of 4 different conductances in every grid point depending on the cells used to calculate this average. For instance, if we want to calculate all the conductances of the pool cell G15 we will use the next commands

Conductance D:           =HARMONIC.AVERAGE(k!G15;k!G16)

Conductance G:           =HARMONIC.AVERAGE(k!G15;k!G14)

Conductance L:           =HARMONIC.AVERAGE(k!F15;k!G15)

Conductance P :           =HARMONIC.AVERAGE(k!G15;k!H15)

However, in the cells that lead to the boundary the specification is difference because they should include the whole term between the pool cell and the environment under the Boundary Condition exigencies. For the Second Kind of Boundary Condition the heat term is directly added to the “ $q$ ” spreadsheet but for both First and Third kind of boundary condition we use the conductances.

For instance, if the border is on the left and we have Third Kind of Boundary Condition along the left surface, the appearance of the conductance  $k_L$  would be:



$$\frac{1}{k_L} = \frac{1}{\alpha \cdot \Delta y \cdot 1} + \frac{\Delta x}{2\lambda_i \Delta y \cdot 1} \quad (2.13)$$

The example below shows the performance of the conductance  $k_L$  belonging to a cell of the piece (F15) that shares the border with an outside fluid with a specific  $\alpha$  (outside cell E15):

$$k_L = 1 / (1/2/k!F15 + 1/k!E15/k!\$C\$4) \quad (2.14)$$

where:

k!F15 is the conductivity in the cell F15

k!E15 is  $\alpha$  (convection coefficient)

k!\\$C\\$4 is  $\Delta y$

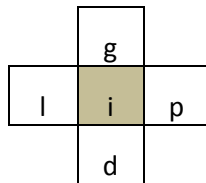
The expression is the same as (2.13) but in our case  $\Delta x = \Delta y$ .

## 2.4 Temperature and Iterative Calculs

From the expression found in chapter 2.1 to determine temperature for every single cell:

$$T_i = \frac{\sum_{j=1}^4 k_{ji} T_j + \dot{q}_{Vi} \Delta x \Delta y}{\sum_{j=1}^4 k_{ji}} \quad (2.15)$$

It is simple to express it with the conductance using the graphic below:



$$T_i = \frac{T_l \cdot k_{Li} + T_g \cdot k_{Gi} + T_p \cdot k_{Pi} + T_d \cdot k_{Di} + \dot{q}_{Vi} \Delta x \Delta y}{k_{Li} + k_{Gi} + k_{Pi} + k_{Di}} \quad (2.16)$$

As is logical in Numerical Methods the temperature of one cell depends on the other cells temperatures. Therefore, the iterative calculations must exist. MS EXCEL provides a

method to use iterative calculations in the way of using it is quite simple. The only thing the user must do before starting is allow the program to make iterations by defining how many of them are needed, the maximum change between the values of the results, and the way MS EXCEL calculates: Automatically or manual.

During the accomplishment of this project the option used it has been *Manual* in order to calculate whenever the user wants by running calculation with the button F9. The user should follow the next MS EXCEL steps to select the explained options,

Click the **Microsoft Office Button** , click **EXCEL Options**, and then click the **Formulas** category.

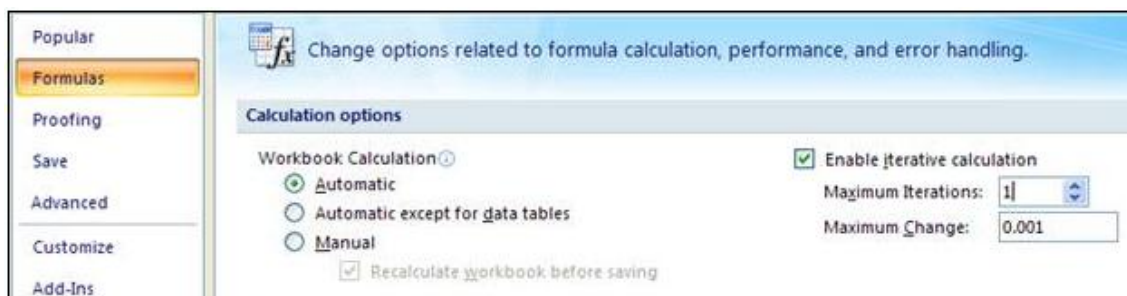
In the **Calculation options section**, under **Workbook Calculation**, clicks **Manual**.

On the right the user may find the option **Enable Iterative Calculations** and the possibility of select **Maximum Number of Iteration** as well as **Maximum Change**.

The **Maximum Change** in temperature should be 0 if the user does not want to have any inaccuracy in the temperature calculation.

During this project the number of iterations has been chosen arbitrary 100 for a first checking and after that cycles of 2000 iterations.

Figure 6. Iterations options in MS EXCEL

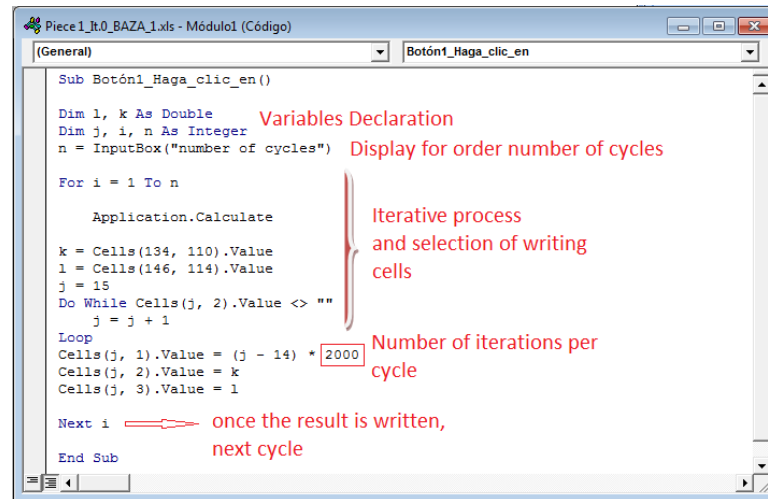


### 2.4.1 Usage of Macros

MS EXCEL with MS VISUAL BASIC provides a method to performance the iterative calculations easily and a way to organize the results by the usage of macros. It consists in an informatics iterative program able to prepare every cycle of iterations and present the results run by MS EXCEL in a table.

In a general view, the macro requires the declaration of variables, the display to order the number of cycles wished, the iterative process responsible to fill the table at every cycle calculated, the writing value cells and the location where the macro will lead the results. Here below the steps may be seen in the macro used for this project.

Figure 7. Macro design



Each time we select the option macros, the program asks how many cycles and write the information as in the next example:

Figure 8. Results of the iterations addressed by macro 1

	A	B	C
8			
9			
10		Macro	
11		1	
12			
13			
14	No	Qd	Qw
15	2000	14453,677457	-249,741655
16	4000	8003,559630	290,477118
17	6000	5713,187507	666,774947
18	8000	4440,527547	932,627266
19	10000	3597,748740	1121,193129
20	12000	3011,326611	1255,158435
21	14000	2597,974471	1350,438088
22	16000	2305,475555	1418,267814
23	18000	2098,137377	1466,598289
24	20000	1950,992626	1501,063542
25	22000	1846,461462	1525,660590
26	24000	1772,134285	1543,228070
27	26000	1719,237813	1555,783921
28	28000	1681,561929	1564,763996
29	30000	1654,705968	1571,190851
30	32000	1635,548284	1575,793322
31	34000	1621,872369	1579,091297
32	36000	1612,102961	1581,455899
33	38000	1605,119558	1583,152245
34	40000	1600,124478	1584,369856
35	42000	1596,549408	1585,244303

Where *No* is the total number of iterations from the begging, in the example is possible to appreciate 21 cycles of 2000 iterations each other.

## 2.5 Predicting and Converging

A numeric solution is always wide convergent; the numbers of iterations needed to find a converging solution are many. The next paragraphs explain a method to predict that converging result without the whole number of iterations and show one of the main targets carried out in this project

If we consider the results achieved every cycle of iterations ( $N$ ) done, we may find a progression of heat transfer values:

$$Q_{N-1}, Q_N, Q_{N+1}, Q_{N+2}, Q_{N+3} \dots \quad (2.17)$$

where  $Q_N$  corresponds to the heat transfer value in the current cycle of iteration.

We could express this progression with the difference between a concrete value and the one corresponding to the previous cycle of iterations, naming this *difference*  $a$ :

$$a_1 = \dot{Q}_N - \dot{Q}_{N+1} \quad (2.18)$$

$$a_2 = \dot{Q}_{N+1} - \dot{Q}_{N+2} \quad (2.19)$$

$$a_3 = \dot{Q}_{N+2} - \dot{Q}_{N+3} \quad (2.20)$$

Now the progression looks like:

$$a_1, a_2, a_3 \dots \quad (2.21)$$

Notice that if we make the sum of the three firsts terms of the progression named as  $a$  the result would be:

$$a_1 + a_2 + a_3 = \dot{Q}_{N+3} - \dot{Q}_N \quad (2.22)$$

If we make the sum of the infinite terms of a progression may be summarized as:

$$a_1 + a_2 + a_3 + \dots = \dot{Q}_\infty - \dot{Q}_N \quad (2.23)$$

From the previous expression we may write:

$$\dot{Q}_{\infty} = \dot{Q}_N + \sum_{i=1}^{\infty} a_i \quad (2.24)$$

Notice that if we consider the sum as an infinite geometric series it will converge if its common ratio absolute value is less than 1 ( $|r| < 1$ ). Its value could then be computed from the finite sum formula:

$$\sum_{i=1}^{\infty} a_i = \frac{a_1}{1-r} \quad (2.25)$$

$$\text{where } r = \frac{a_{n+1}}{a_n} \quad - \text{ common ratio} \quad (2.26)$$

Being  $r$  the common ratio the successive terms  $a$  have. So that, we could express the current value of  $r$  as:

$$r_N = \frac{\dot{Q}_{N+2} - \dot{Q}_{N+1}}{\dot{Q}_{N+1} - \dot{Q}_N} \quad (2.27)$$

Adding the last expression to the expression (2.24) is possible to predict the value of  $\dot{Q}_{\infty}$  with the current value of heat and its second next cycle's values:

$$\dot{Q}_{\infty} = \dot{Q}_N + \frac{\dot{Q}_{N+1} - \dot{Q}_N}{1 - r_N} \quad (2.28)$$

The next table shows one of the examples studied in this project with the formulas written for a concrete cycle of iterations. In it is possible to find the formula for  $r_N$  corresponding to the cycle  $N = 6$  (with a total number of iterations equal to 12000) and the formula for the prediction of heat transfer ( $Q = \dot{Q}_{\infty}$ ) in the cycle  $N = 13$  (26000 iterations).

Figure 9. Calculation of “r” and “Q”

	A	B	D	J	K	L	M	N
1								
2	N	n	Qw	Qw				
3				%	r	r (%)	Q	Q (%)
4	1	2000	1905,623693					
5	2	4000	1864,998819					
6	3	6000	1835,079807					
7	4	8000	1812,993542		0,7397	2,1000		
8	5	10000	1796,671060	2,6723				
9	6	12000	1784,597314	1,9824	=(D11-D10)/(D10-D9)			0,011006
10	7	14000	1775,658951	1,4716	0,7414	1,8687	1750,0456	0,007880
11	8	16000	1769,036566	1,0931	0,7420	1,7987	1750,0067	0,005655
12	9	18000	1764,126411	0,8125	0,7425	1,7316	1749,9789	0,004066
13	10	20000	1760,483189	0,6043	0,7430	1,6672	1749,9590	0,002930
14	11	22000	1757,778156	0,4498	0,7434	1,6053	1749,9447	0,002115
15	12	24000	1755,768396	0,3349	0,7439	1,5456		
16	13	26000	1754,274263	0,2495	0,7443	1,4881	=D16+(D17-D16)/(1-K16)	
17	14	28000	1753,162794	0,1860	0,7447	1,4324	1749,9218	0,000803
18	15	30000	1752,335500	0,1387	0,7452	1,3786	1749,9179	0,000583
19	16	32000	1751,719378	0,1035	0,7455	1,3265	1749,9151	0,000424
20	17	34000	1751,260274	0,0773	0,7459	1,2760	1749,9131	0,000308

## 2.6 Temperature distribution graphic

Once each case is solved it is possible to draw a graphic with the temperature distribution choosing the colors in a concrete temperature range. MS EXCEL does not draw the graphic automatically but is possible to set the proper options to achieve a coherent graphic with the typical colors used for temperatures. Always is recommended to begin with red for the highest temperatures and finish with blue for the lowest temperatures, going by orange, yellow and green in this order.

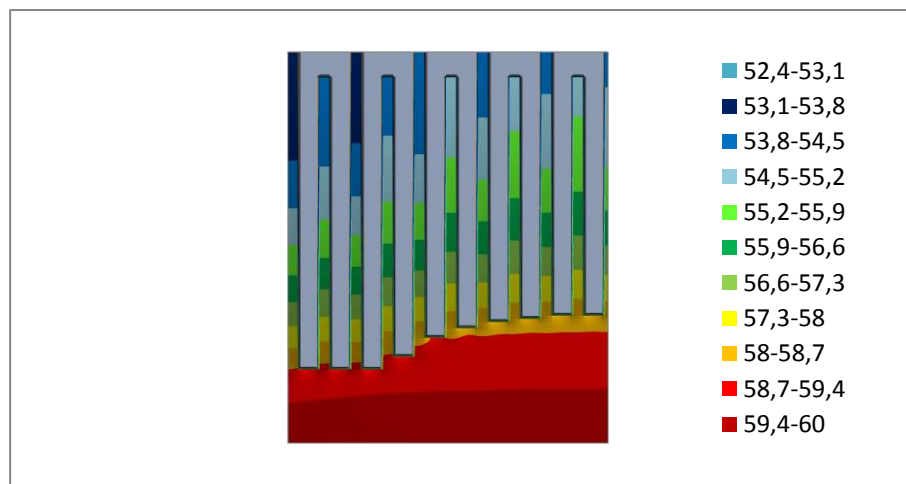
The sort of graphic needed is *Surface Graphic* and after selecting the data (the whole geometry of the piece with its temperatures calculated) the user can proceed to decide which options use.

In this project the aim was present a graphic as much similar as possible to the ones that provides programs like ANSYS. The needed options that have been modified are:

- Range of temperatures. Always observing from the *Temperature Sheet* the maximum and the minimum that the piece presents.
- Format – Colors. This step is manual. Once the chart legend is shown beside the graphic the user can modify manually each color that appears in it by selecting its range of temperatures and choosing the proper color.

- Change between temperatures. The user can also select the range of temperatures belonging to each color, for example, every  $1^{\circ}\text{C}$  the color must be changed. Always depends on total range of temperatures the piece has. Because of this, every single test made in this project has its proper range values and its number of different colors.

Figure 10. Example of temperature distribution graphic



In the graphic above a temperature distribution graphic is presented. As it is been remarked the colors change from red to blue as they look for cooler temperatures.

In this case the total range of temperatures was from  $53,1^{\circ}\text{C}$  to  $60^{\circ}\text{C}$  changing every  $0,7^{\circ}\text{C}$ .

#### Reference used in chapter 2:

[3] Zdzisław Nagórski, *“Modelowanie przewodzenia ciepła za pomocą arkusza kalkulacyjnego”*, Warszawa 2001. (in polish)

[4] Sucheta A., Sikora K., *“Solving two dimensional steady heat conduction with the use of spreadsheet”*, XII Symposium on Heat and Mass Transfer, AGH-Kraków 2004. p.795-804 (in polish)

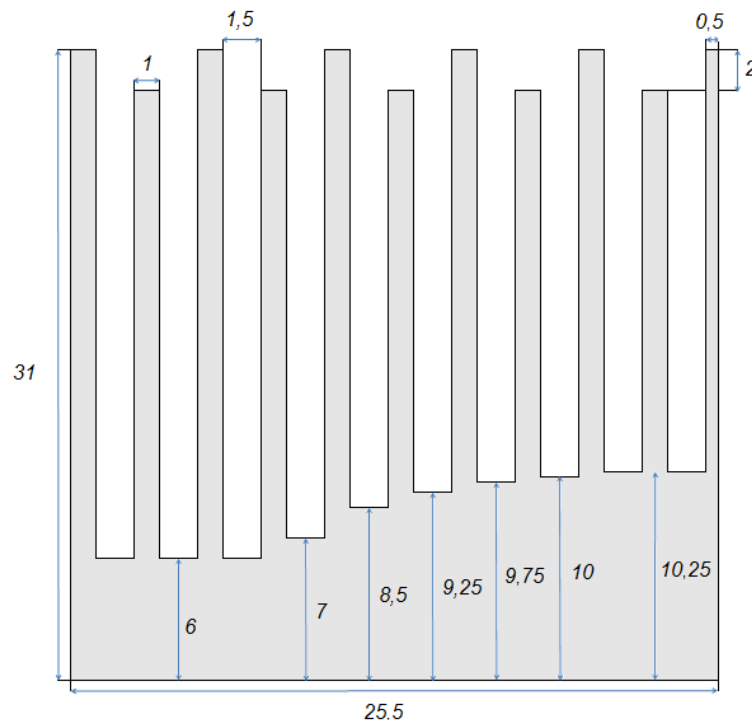
## 3 Experimental Cases

### 3.1 Modeling

The first step is the description of the piece geometry. Nowadays, MS EXCEL does not have a specialized drawing option like other programs, so the aim is the usage of the cells in order to create the contour of the piece. May notice that the selection of the cells for the contour will also affect on the final discretization of the body, whose grid points will be determined by the total number of cells found inside the contour. Once the size of the cell is determined and the dimensions of the piece are given is possible to draw the piece.

In the next draft are indicated the piece dimensions given, all the measures are expressed in millimeters. The depth is considered as a unit.

Figure 11. Piece measures and geometry



The draft has been carried out selecting for every measure the correct number of cells needed, knowing that the discretization required for both length and height was 0,25 mm. That means that every single cell represents a perfect square of side 0,25 mm. For instance, the side whose measure is 31 has needed 124 cells. The cells are not measured but its size value is specified in the MS EXCEL sheet for next calculations.



Figure 12. Cells representing size

$\Delta x = \Delta y$	0,00025 m
$\lambda$	200 W/mK
$\alpha$	100 W/m <sup>2</sup> K

Once the geometry has been defined, the next step is the match of the different conductance explained in the chapter 2.2. For every test that has been made the only changing values are the conductivity and the convection coefficient, in addition of the changes in the boundary conditions. On the contrary, the setting of the conductances (P, D, G, L) is not needed to change, thus, every different test does not require a reset of all the values.

As is explained in chapter 2 the conductance corresponding to a contour cell of the piece has been written as the inverse of the resistance between the body and the surrounding, taking into account the boundary condition given along the area. By this procedure, is possible to express the heat flux in the boundary as the following general equation for every border cell:

$$\dot{Q}_{boundary} = conductance (P, D, G \text{ or } L) \cdot (T_s - T_i) \quad (3.1)$$

Where  $T_s$  represents the temperature in the surrounding and  $T_i$  represents the temperature of a specific cell next to it.

On the other hand, the selection of the constant (P, D, G, L) depends on the position of the cell, following the indications given in chapter 2.

Therefore, the constants next to the border will have the expression that the boundary condition requires.

With the constants defined, the geometry set, and the heat transfer equation introduced along the surrounding is possible to prepare temperature equation in order to find converging results and the possibility of predicting it. Recovering the information given in chapter 2.4 every single cell belonging to the piece will have this expression in the temperature spreadsheet

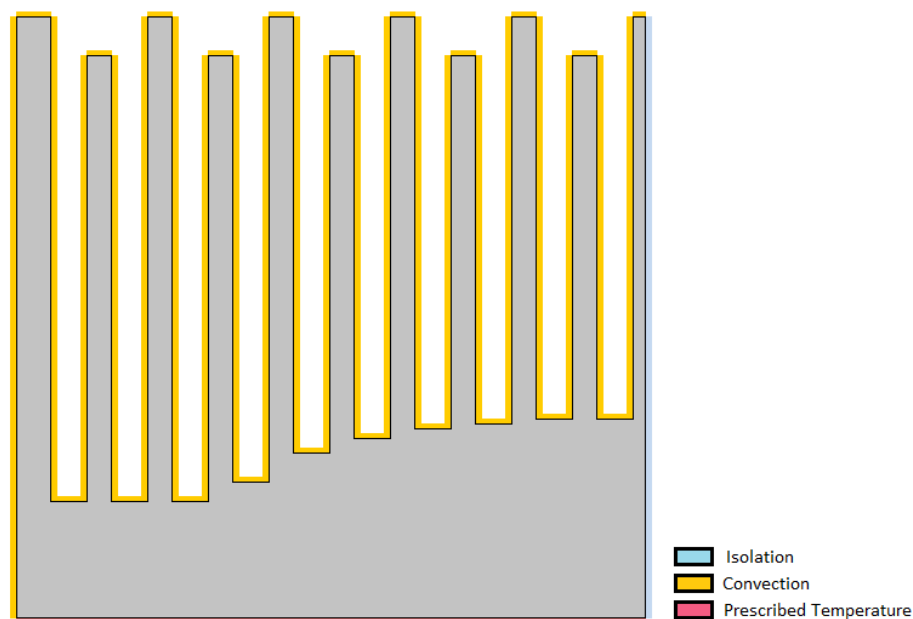
$$T_i = \frac{T_l \cdot k_{Li} + T_g \cdot k_{Gi} + T_p \cdot k_{Pi} + T_d \cdot k_{Di} + \dot{q}_{Vi} \Delta x \Delta y}{k_{Li} + k_{Gi} + k_{Pi} + k_{Di}} \quad (3.2)$$

Eventually, every single test whose results will be indicated in the next chapters is performance with isolation in the whole right border of the piece. So that, the conductance  $k_p$  in this border is equal to zero.

### 3.2 First and Third kind of Boundary Condition

In the firsts tests made for the three material studied (Aluminum, Cooper and Steel) and for both natural and forced convection by the setting of different conductivities and convection coefficient, the heat flux crosses the piece from the base, which is at specific temperature (First Kind on Boundary Condition) and is dissipated along the fins by convection (Third Kind of Boundary Conditions). The figure below shows the boundary conditions of the piece:

Figure 13. Isolation, convection and prescribed temperature surrounding the piece



#### **Prescribed Temperature in the base ( $Q_d$ ) First Kind of Boundary Condition**

As has been explained in previous paragraphs, the First Kind of Boundary condition corresponds to a prescribed temperature in the surrounding. In this first case, the fixed temperature has been considered 333 K (60° C).

The graphics below show the set temperature, the conductance related under this boundary conditions and the expression of heat flux crossing the base surface.

Figure 14. Conductance and prescribed temperature

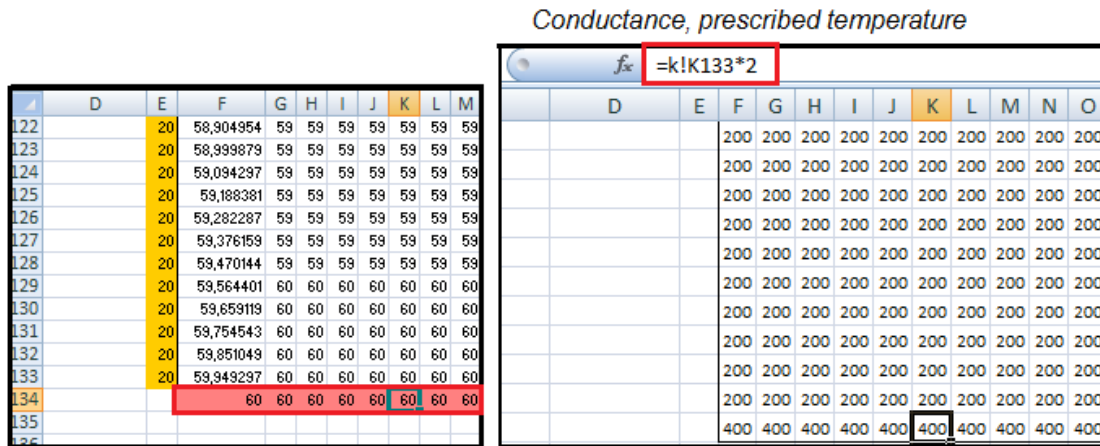
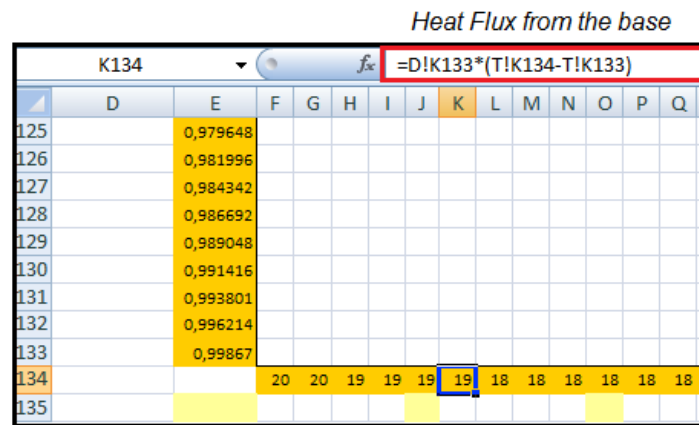


Figure 15. Heat flux equation from the piece base



Notice that the expression of the conductance in the base is derived from First Kind of Boundary Conditions:

$$\frac{1}{k_D} = \frac{\Delta x}{\Delta y} \cdot \frac{1}{2\lambda_i} = \frac{1}{2\lambda_i} \text{ for } \Delta x = \Delta y \quad (3.3)$$

So that, we can apply:

$$k_D = 2\lambda_i \quad (3.4)$$

That corresponds with the expression fit in MS EXCEL, and shown in the last graphic: “k!K133\*2” where K133 is  $\lambda_i$ .

The determination of heat transfer from the base cell is calculated by multiplying  $k_D$  with the temperature difference between the cell and the given temperature of  $60^\circ$ .

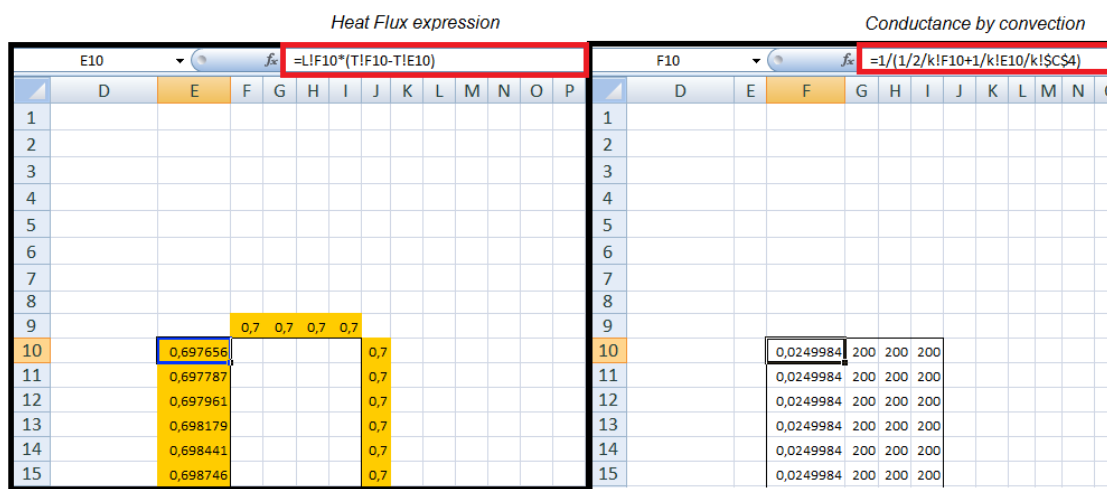
### Heat flux across the fins ( $Q_w$ ). Third Kind of Boundary Condition

The way it is determined the heat flux across the finned surface to the environment is described by the equation (3.1). The only difference with the last paragraphs performance lies in the description of convection in the fins border. The conductance (L, G and P) are determined with the next expression already shown in chapter 2:

$$\frac{1}{k} = \frac{1}{\alpha \cdot \Delta y \cdot 1} + \frac{\Delta x}{2 \cdot \lambda \cdot \Delta y \cdot 1} \quad (3.5)$$

These conductances are used depending on the direction the heat is fluxing. Next graphic shows an example of dissipation by convection in one of the fins tip of the piece:

Figure 16. Heat flux and conductance for convection



### 3.2.1 Forced Convection ( $\alpha = 100$ )

#### Aluminum

Considering a conductivity of  $\lambda_{Al} = 200 \frac{W}{mK}$ , it has been carried out the first test of the project with aluminum. In order to achieve results as fast as possible it has been used the macros designed and described in chapter 2.4.1. The program has executed 79-cycles of 2000 iterations. We have run a lot of iterations in order to check that the formulas were correct by observing that the values of heat transfer ( $Q_w$  and  $Q_d$ ) converge. In previous calculations the numbers were not converging so it warned that a mistake had occurred during the modeling.

In the table below are shown the results of the heat transfer in aluminum as well as the determination of the coefficient  $r$  and the predicting value  $Q$ . In there are marked the

significant numbers from where the prediction can be made with enough accuracy, approaching the value of  $r$  until the hundredths

Table 2. Aluminum, forced convection. Results

n	Qd	Qw	Qd				Qw					
			%	r	r (%)	Q	Q (%)	%	r	r (%)	Q	Q (%)
2000	14453,6775	-249,7417										
4000	8003,5596	290,4771										
6000	5713,1875	666,7749										
8000	4440,5275	932,6273		0,6958	3,9959				0,7104	2,6141		
10000	3597,7487	1121,1931	126,6318	0,7049	2,7471	1610,7439		29,3728	0,7112	2,5067	1585,1042	
12000	3011,3266	1255,1584	89,6915	0,7076	2,3669	1597,5456	0,6337	20,9340	0,7119	2,4142	1585,8774	0,1013
14000	2597,9745	1350,4381	63,6534	0,7089	2,1979	1593,3376	0,3686	14,9320	0,7125	2,3285	1586,3893	0,0691
16000	2305,4756	1418,2678	45,2281	0,7097	2,0829	1591,2926	0,2398	10,6593	0,7131	2,2476	1586,7350	0,0473
18000	2098,1374	1466,5983	32,1673	0,7104	1,9847	1590,0464	0,1613	7,6148	0,7137	2,1708	1586,9701	0,0325
20000	1950,9926	1501,0635	22,8982	0,7111	1,8942	1589,2270	0,1097	5,4437	0,7142	2,0976	1587,1307	0,0224
22000	1846,4615	1525,6606	16,3135	0,7117	1,8089	1588,6758	0,0749	3,8943	0,7147	2,0276	1587,2407	0,0155
24000	1772,1343	1543,2281	11,6315	0,7123	1,7280	1588,3019	0,0514	2,7877	0,7152	1,9606	1587,3162	0,0107
26000	1719,2378	1555,7839	8,2994	0,7128	1,6509	1588,0472	0,0353	1,9967	0,7157	1,8963	1587,3683	0,0074
28000	1681,5619	1564,7640	5,9261	0,7133	1,5773	1587,8731	0,0244	1,4310	0,7161	1,8344	1587,4042	0,0052
30000	1654,7060	1571,1909	4,2344	0,7139	1,5068	1587,7537	0,0169	1,0262	0,7166	1,7747	1587,4291	0,0036
32000	1635,5483	1575,7933	3,0276	0,7144	1,4391	1587,6716	0,0117	0,7363	0,7170	1,7172	1587,4464	0,0025
34000	1621,8724	1579,0913	2,1661	0,7148	1,3739	1587,6150	0,0081	0,5285	0,7174	1,6615	1587,4584	0,0018
36000	1612,1030	1581,4559	1,5507	0,7153	1,3111	1587,5758	0,0056	0,3796	0,7178	1,6077	1587,4667	0,0012
38000	1605,1196	1583,1522	1,1108	0,7157	1,2505	1587,5487	0,0039	0,2727	0,7182	1,5555	1587,4726	0,0009
40000	1600,1245	1584,3699	0,7961	0,7161	1,1918	1587,5298	0,0027	0,1960	0,7185	1,5048	1587,4766	0,0006
42000	1596,5494	1585,2443	0,5709	0,7166	1,1349	1587,5167	0,0019	0,1409	0,7189	1,4556	1587,4795	0,0004
44000	1593,9891	1585,8726	0,4096	0,7170	1,0797	1587,5076	0,0013	0,1014	0,7192	1,4078	1587,4815	0,0003
46000	1592,1546	1586,3243	0,2941	0,7173	1,0261	1587,5012	0,0009	0,0729	0,7196	1,3612	1587,4829	0,0002
48000	1590,8393	1586,6492	0,2112	0,7177	0,9740	1587,4967	0,0007	0,0524	0,7199	1,3160	1587,4839	0,0001
50000	1589,8957	1586,8830	0,1518	0,7181	0,9233	1587,4936	0,0005	0,0377	0,7202	1,2719	1587,4846	0,0001
52000	1589,2185	1587,0513	0,1091	0,7184	0,8739	1587,4914	0,0003	0,0271	0,7205	1,2289	1587,4850	0,0001
54000	1588,7323	1587,1725	0,0785	0,7188	0,8257	1587,4899	0,0002	0,0195	0,7209	1,1871	1587,4854	0,0001
56000	1588,3829	1587,2598	0,0565	0,7191	0,7788	1587,4888	0,0002	0,0140	0,7212	1,1463	1587,4856	0,0000
58000	1588,1318	1587,3228	0,0407	0,7195	0,7329	1587,4880	0,0001	0,0100	0,7214	1,1065	1587,4858	0,0000
60000	1587,9512	1587,3682	0,0293	0,7198	0,6882	1587,4875	0,0001	0,0071	0,7217	1,0678	1587,4859	0,0000
62000	1587,8212	1587,4010	0,0211	0,7201	0,6445	1587,4871	0,0001	0,0051	0,7220	1,0300	1587,4860	0,0000
156000	1587,4862	1587,4862										
158000	1587,4862	1587,4862										

The results show a clear convergence between the values of  $Q_w$  and  $Q_d$ . For aluminum the total heat that crosses the piece and is dissipated by convection is:

$$Q_{Al} = 1587,4862 \frac{W}{m}$$

Thus, this is the heat flux that a piece of aluminum of the specified geometry is able to dissipate using a finned surface and taking into account that the temperature in the base is fixed and equal to  $60^\circ$ .

It is important to comment that in this first case no temperatures were established before, so that the program started running without any reference in the temperature sheet but its formulas. Anyway, in the next cases may notice that the number iterations needed to converge, in spite of starting points with temperature approached to the final ones, is quite similar to the table above.

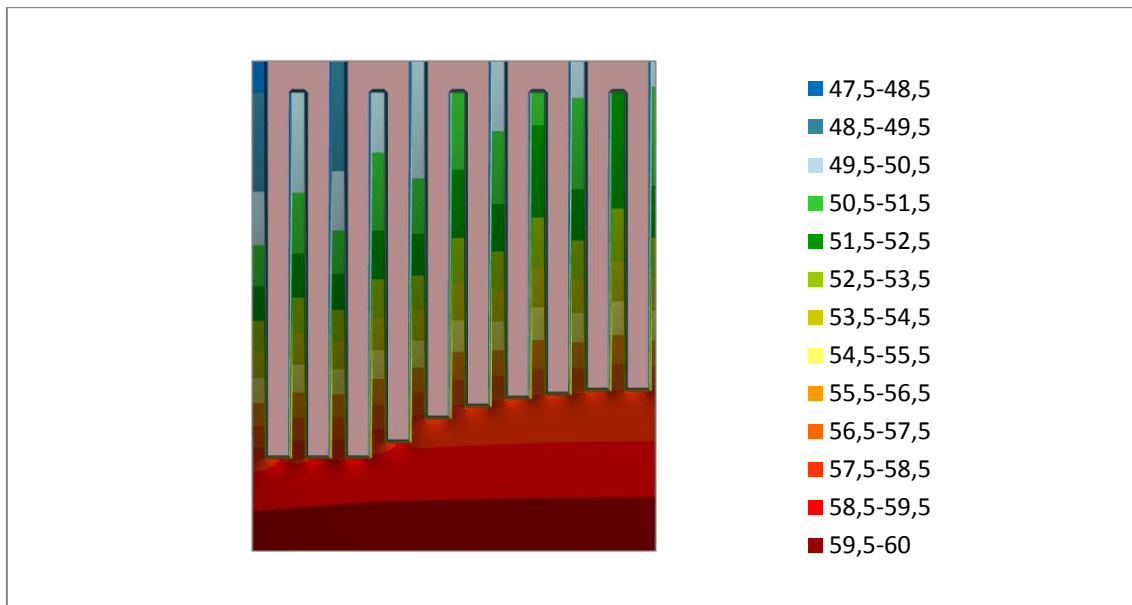
It has been marked in colors blue and salmon the number of iterations from where the prediction is quite close to the final values. For  $Q_d$  are need 62000 iterations and for  $Q_w$  50000. In both cases the mistake done by the prediction is tiny: 0,0001%.

The timing needed for getting the values of heat depends on the computer used to accomplish it, but generally takes around one minute per 2000 iterations.

After checking that the heat transfer has succeed in its convergence is possible to make a graphic with the temperature distribution, as it will be commented in the chapter 6.1 (comparison between ANSYS and EXCEL), MS EXCEL requires a slower way to organize the information requested for the graphic, for instance, the selection of the colors to show the areas with high and low temperature and distinguish them.

In the next graphic is possible to have a look to the temperature distribution found for the aluminum piece:

Figure 17. Aluminum, forced convection. Temperature distribution graphic



As is shown the temperature rise from  $47,5^{\circ}$  more or less, until close to  $60^{\circ}$  near to the base. As it will be notice in all tests, the fin which temperature is cooler is the vey left one. This fact is related with the isolation situation of the right surface and also with the widest dissipation area belonging to the left fin.

In the next example, cooper, the dissipation should be bigger due to the highest value of conductivity, thus, once the aluminum test was finished we were expecting highest values of heat transfer and higher temperatures in the fins.

### Cooper

In order to use the last modeling for aluminum but with a starting point for iterations different than the solution found for the aluminum case, we brought the piece until a uniform distribution temperature equal to  $60^{\circ}$ . From this starting point the iterations would run the calculations from a temperature different than the expected one.

By the setting of  $\alpha = 1000 \frac{W}{m^2 K}$  and the whole surrounding temperature of  $60^{\circ}C$  is possible to achieve in a low number of iterations an obviously total amount of heat flux equal to zero (no temperature difference between base and environment) and the wanted uniform temperature distribution. Thus, the wished started point was reached easily and was possible to carry on with the next test. The table below shows the results of the iterations for Cooper.

Table 3. Cooper, forced convection. Results

n	Qd	Qw	Qd				Qw					
			%	r	r(%)	Q	Q (%)	%	r	r (%)	Q	Q (%)
2000	447,501050	1905,623693										
4000	799,298554	1864,998819										
6000	1050,878143	1835,079807										
8000	1234,646752	1812,993542		0,7360	2,4982				0,7397	2,1000		
10000	1369,636878	1796,671060	21,7309	0,7369	2,3806	1747,3245		2,6723	0,7403	2,0188	1750,1774	
12000	1468,995275	1784,597314	16,0530	0,7377	2,2837	1748,1008	0,103255	1,9824	0,7409	1,9420	1750,1003	0,011006
14000	1542,215426	1775,658951	11,8688	0,7383	2,1949	1748,6283	0,073111	1,4716	0,7414	1,8687	1750,0456	0,007880
16000	1596,227108	1769,036566	8,7822	0,7390	2,1112	1748,9973	0,052023	1,0931	0,7420	1,7987	1750,0067	0,005655
18000	1636,105653	1764,126411	6,5033	0,7396	2,0316	1749,2579	0,037134	0,8125	0,7425	1,7316	1749,9789	0,004066
20000	1665,574457	1760,483189	4,8193	0,7401	1,9556	1749,4426	0,026576	0,6043	0,7430	1,6672	1749,9590	0,002930
22000	1687,368552	1757,778156	3,5739	0,7407	1,8828	1749,5741	0,019065	0,4498	0,7434	1,6053	1749,9447	0,002115
24000	1703,499204	1755,768396	2,6521	0,7412	1,8131	1749,6678	0,013707	0,3349	0,7439	1,5456	1749,9345	0,001529
26000	1715,446981	1754,274263	1,9693	0,7417	1,7460	1749,7349	0,009875	0,2495	0,7443	1,4881	1749,9271	0,001107
28000	1724,302845	1753,162794	1,4632	0,7422	1,6815	1749,7830	0,007127	0,1860	0,7447	1,4324	1749,9218	0,000803
30000	1730,871421	1752,335500	1,0878	0,7427	1,6192	1749,8175	0,005153	0,1387	0,7452	1,3786	1749,9179	0,000583
32000	1735,746670	1751,719378	0,8092	0,7431	1,5590	1749,8424	0,003731	0,1035	0,7455	1,3265	1749,9151	0,000424
34000	1739,367411	1751,260274	0,6023	0,7436	1,5008	1749,8603	0,002706	0,0773	0,7459	1,2760	1749,9131	0,000308
36000	1742,058099	1750,917991	0,4486	0,7440	1,4445	1749,8733	0,001964	0,0577	0,7463	1,2270	1749,9116	0,000224
38000	1744,058818	1750,662673	0,3342	0,7444	1,3898	1749,8827	0,001428	0,0431	0,7467	1,1795	1749,9106	0,000163
40000	1745,547346	1750,472131	0,2492	0,7448	1,3367	1749,8895	0,001039	0,0323	0,7470	1,1333	1749,9098	0,000119
42000	1746,655422	1750,329861	0,1859	0,7452	1,2851	1749,8945	0,000756	0,0241	0,7473	1,0884	1749,9092	0,000087
44000	1747,480728	1750,223584	0,1387	0,7456	1,2350	1749,8980	0,000551	0,0181	0,7477	1,0447	1749,9088	0,000063
46000	1748,095746	1750,144159	0,1035	0,7459	1,1862	1749,9007	0,000402	0,0135	0,7480	1,0022	1749,9085	0,000046
48000	1748,554290	1750,084775	0,0773	0,7463	1,1387	1749,9026	0,000293	0,0101	0,7483	0,9609	1749,9083	0,000034
50000	1748,896339	1750,040356	0,0578	0,7467	1,0925	1749,9039	0,000214	0,0076	0,7486	0,9207	1749,9081	0,000025
52000	1749,151612	1750,007117	0,0432	0,7470	1,0474	1749,9050	0,000156	0,0057	0,7489	0,8816	1749,9080	0,000018
54000	1749,342213	1749,982234	0,0323	0,7473	1,0035	1749,9057	0,000114	0,0043	0,7492	0,8435	1749,9079	0,000013
56000	1749,484590	1749,963599	0,0242	0,7476	0,9607	1749,9062	0,000083	0,0032	0,7495	0,8065	1749,9079	0,000010
58000	1749,590993	1749,949638	0,0181	0,7480	0,9190	1749,9066	0,000061	0,0024	0,7497	0,7704	1749,9078	0,000007
60000	1749,670544	1749,939174	0,0136	0,7483	0,8783	1749,9069	0,000044	0,0018	0,7500	0,7353	1749,9078	0,000005
62000	1749,730046	1749,931329	0,0102	0,7486	0,8387	1749,9071	0,000032	0,0014	0,7503	0,7011	1749,9078	0,000004
64000	1749,774570	1749,925445	0,0076	0,7489	0,8000	1749,9073	0,000023	0,0010	0,7505	0,6679	1749,9078	0,000003
66000	1749,807899	1749,921031	0,0057	0,7491	0,7624	1749,9074	0,000017	0,0008	0,7508	0,6356	1749,9077	0,000002
68000	1749,832857	1749,917717	0,0043	0,7494	0,7256	1749,9075	0,000012	0,0006	0,7510	0,6041	1749,9077	0,000001
70000	1749,851555	1749,915230	0,0032	0,7497	0,6898	1749,9075	0,000009	0,0004	0,7512	0,5735	1749,9077	0,000001
72000	1749,865568	1749,913362	0,0024	0,7500	0,6550	1749,9076	0,000006	0,0003	0,7515	0,5438	1749,9077	0,000001
128000	1749,907687	1749,907704										
130000	1749,90769	1749,907703										

The number of iterations required to achieve the comparing values are:

- 18000 for getting an error lower than 0.05% in  $Q_d$  and lower than 0,005% in  $Q_w$ .



- 60000 for  $Q_w$  to approach until  $r_w = 0,75$ . That shows an error in the prediction of almost 0 % taking into account all the decimals
- 72000 for  $Q_d$  to approach until  $r_d = 0,75$ . The error is also minimum.

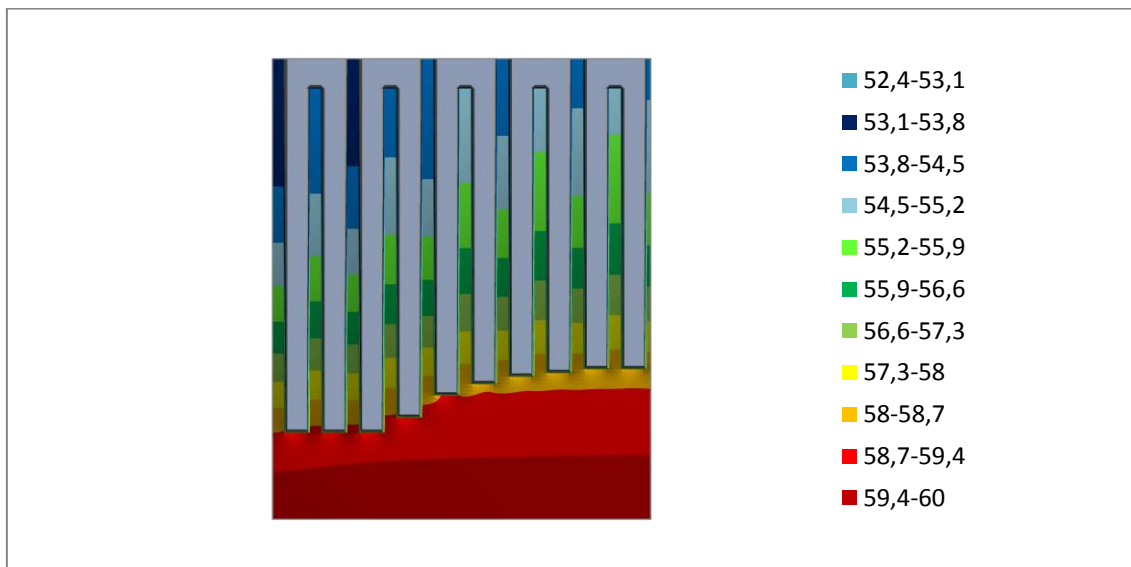
$$Q_{Cu} = 1749,9077 \frac{W}{m}$$

- In order to get an error of 0,0001% (like in the aluminum test) are needed 38000 iterations for  $Q_w$  and 50000 iterations for  $Q_d$ .
- Number of iterations to get final results without predicting was about 130000.

However, may notice that from the beginning the predicting values are quite similar to the final ones due to the starting point. On the contrary, the test with aluminum needed more to reach amounts of heat similar to the actual values because it had not any reference at the moment the iterations run.

The temperature distribution is found in the next graphic. Now, the coldest temperatures (also situated in the left fin tips) are higher than before. The conductivity of cooper, considered as  $\lambda_{Cu} = 395 \frac{W}{mK}$  is almost the double value of the aluminum's. Therefore, the heat dissipated is higher without needing a lower temperature in the fin tips thanks to the better conductivity among the body. However, the isotherms are quite similar to the temperature distribution in aluminum due to the kind of heat transferred. Once again the coldest temperatures are situated in the left fin tip.

Figure 18. Cooper, forced convection. Temperature distribution graphic





## Steel

The material with less conductivity ( $\lambda_{Steel} = 45 \frac{W}{mK}$ ) tested in this project is the steel. If the conductivity is lower than in the previous cases, consequently, the dissipation would be worse; the material has not the same easiness to conduct heat throughout its dominium.

The next table shows the iterations needed to achieve convergence. The point of this case is the possibility of comparing it with the previous one (cooper), because both of them have started iterations from the same conditions: temperature uniform distribution of 60°C throughout the piece.

Table 4. Steel, forced convection. Results

n	Qd	Qw	Qd					Qw				
			%	r	r(%)	Q	Q(%)	%	r	r(%)	Q	Q(%)
2000	414,5297	1528,1118										
4000	677,7874	1293,1697										
6000	821,6878	1162,9283										
8000	901,0850	1090,5626		0,5539					0,5566			
10000	945,0113	1050,3132	5,4698	0,5545	-9,3755	999,6277		-5,0636	0,5570	-0,3550	999,7389	
12000	969,3443	1027,9093	3,0357	0,5549	-9,4669	999,6592	0,003318	-2,8226	0,5574	-0,4182	999,7166	0,002419
14000	982,8363	1015,4301	1,6861	0,5554	-9,5505	999,6752	0,001715	-1,5743	0,5577	-0,4767	999,7050	0,001265
16000	990,3235	1008,4747	0,9372	0,5558	-9,6276	999,6835	0,000891	-0,8785	0,5580	-0,5312	999,6990	0,000665
18000	994,4816	1004,5957	0,5212	0,5561	-9,6993	999,6877	0,000466	-0,4905	0,5583	-0,5822	999,6959	0,000351
20000	996,7925	1002,4313	0,2901	0,5565	-9,7661	999,6899	0,000244	-0,2740	0,5585	-0,6301	999,6942	0,000186
22000	998,0777	1001,2230	0,1615	0,5568	-9,8287	999,6911	0,000129	-0,1531	0,5588	-0,6754	999,6934	0,000099
24000	998,7928	1000,5481	0,0900	0,5571	-9,8877	999,6917	0,000068	-0,0856	0,5590	-0,7184	999,6929	0,000053
26000	999,1909	1000,1710	0,0502	0,5574	-9,9434	999,6920	0,000036	-0,0479	0,5593	-0,7592	999,6927	0,000028
28000	999,4127	999,9602	0,0280	0,5576	-9,9962	999,6922	0,000019	-0,0268	0,5595	-0,7983	999,6925	0,000015
30000	999,5364	999,8423	0,0156	0,5579	-10,0464	999,6923	0,000010	-0,0150	0,5597	-0,8357	999,6925	0,000008
32000	999,6053	999,7763	0,0087	0,5581	-10,0944	999,6923	0,000006	-0,0084	0,5599	-0,8716	999,6924	0,000004
34000	999,6437	999,7394	0,0049	0,5584	-10,1404	999,6923	0,000003	-0,0047	0,5601	-0,9061	999,6924	0,000002
36000	999,6652	999,7187	0,0027	0,5586	-10,1845	999,6924	0,000002	-0,0026	0,5603	-0,9394	999,6924	0,000001
38000	999,6772	999,7071	0,0015	0,5588	-10,2270	999,6924	0,000001	-0,0015	0,5604	-0,9716	999,6924	0,000001
40000	999,6839	999,7006	0,0008	0,5590	-10,2680	999,6924	0,000000	-0,0008	0,5606	-1,0028	999,6924	0,000000
42000	999,6876	999,6970	0,0005	0,5592	-10,3076	999,6924	0,000000	-0,0005	0,5608	-1,0329	999,6924	0,000000
44000	999,6897	999,6950	0,0003	0,5594	-10,3459	999,6924	0,000000	-0,0003	0,5609	-1,0622	999,6924	0,000000
46000	999,6909	999,6938	0,0001	0,5596	-10,3831	999,6924	0,000000	-0,0001	0,5611	-1,0906	999,6924	0,000000
48000	999,6915	999,6932	0,0001	0,5598	-10,4192	999,6924	0,000000	-0,0001	0,5612	-1,1183	999,6924	0,000000
50000	999,6919	999,6928	0,0000	0,5599	-10,4544	999,6924	0,000000	0,0000	0,5614	-1,1452	999,6924	0,000000
90000	999,692371	999,692371										
92000	999,692371	999,692371										

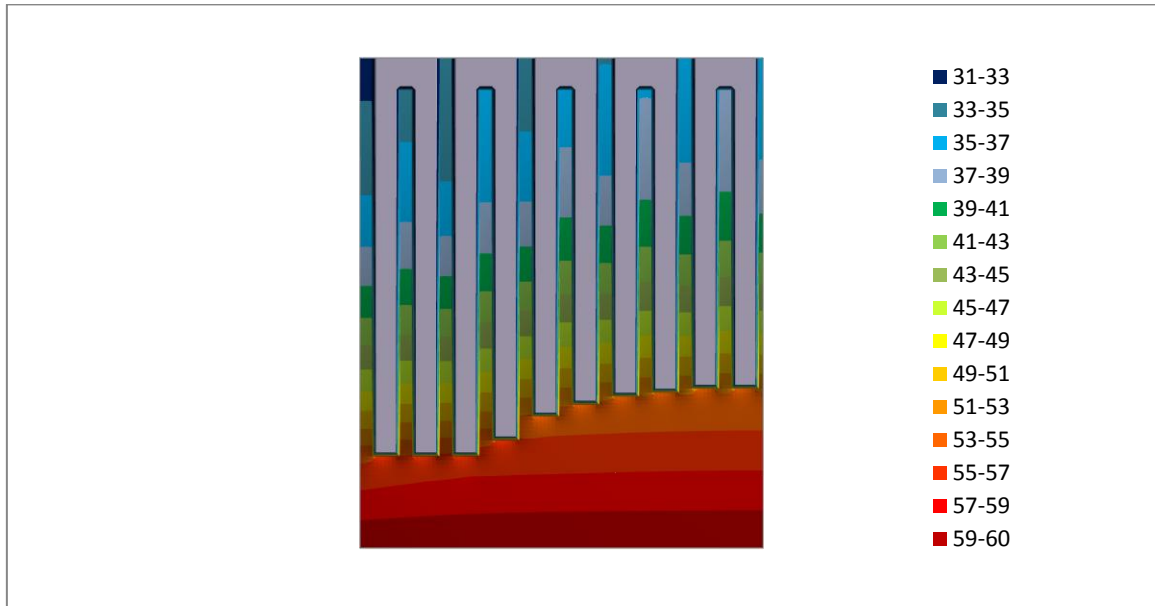
In this case the convergence is easily reached. The conductivity is not enough to keep on transferring heat and becomes quickly the steady temperature. From the first iterations the values get quite similar to the final ones.

The total amount of heat dissipated is, as we have guessed, quite lower than in the previous tests made:

$$Q_{Steel} = 999,6923 \frac{W}{m}$$

The temperature distribution shown in the graph in the next page, indicates a coldest temperature of almost 31°C, due to the worse conduction but same convection in the fins as the previous examples.

Figure 19. Steel, forced convection. Temperature distribution graphic



The isotherm pattern remains the same but the range of temperatures is the widest. The steel is not able to conduct the heat in such as good level compared with Aluminum or Cooper. The fact of achieve these temperatures would be also an important data to evaluate the fin efficiency in coming chapters.

### 3.2.2 Natural Convection

#### Determination of natural convection coefficient: $\alpha$

In general, heat transfer relations in natural convection are based in experimental cases.

Here below we show the simple correlations used to determine the natural convection [5]:

The first step carried out is the determination of Rayleigh number with the next expression:

$$Ra = Gr \cdot Pr = \frac{g \cdot \beta \cdot (T_s - T_\infty) \cdot \delta^3}{\nu^2} \quad (3.6)$$

where

Gr = Grahof number

Pr = Prandtl number

$g$  = gravity = 9,81 m/s<sup>2</sup>

$T_s = 60^\circ\text{C}$

$T_\infty = 20^\circ\text{C}$

$\delta$  = base height =  $2 \cdot 0.0255 = 0.051 \text{ m}$ .

The properties of air at the film temperature  $T_f = \frac{T_s + T_\infty}{2} = 40^\circ\text{C} = 313 \text{ K}$  and 1 atm pressure are:

$$\beta = \frac{1}{T_f} = 0,00319 \text{ K}^{-1} \quad v = \text{kinematic viscosity} = 1,74 \cdot 10^{-5} \frac{\text{m}^2}{\text{s}}$$

$$\text{Pr} = 0.716 \quad \lambda_f = \text{Thermal Conductivity of the fluid} = 0,027 \frac{\text{W}}{\text{m} \cdot \text{K}}$$

The result for the Rayleigh number applying all the values is:

$$\text{Ra} = 407863,31$$

Then, the natural convection Nusselt number in this case can be determined with the expression for free convection in a vertical wall:

$$\text{Nu} = 0,6 + \left\{ \frac{0,387 \text{ Ra}^{1/6}}{[1 + (0,559/\text{Pr})^{9/16}]^{8/27}} \right\}^2 = 11,336 \quad (3.7)$$

The natural convection coefficient is found out from the next expression:

$$\alpha = \frac{\lambda_f}{\delta} \text{Nu} = 6 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad (3.8)$$

In the next paragraphs are presented the results achieved for natural convections using the three materials selected. All the tests have started from a uniform temperature of 60° overall the piece. Afterwards, a brief conclusion comparing forced and natural convection is explained.

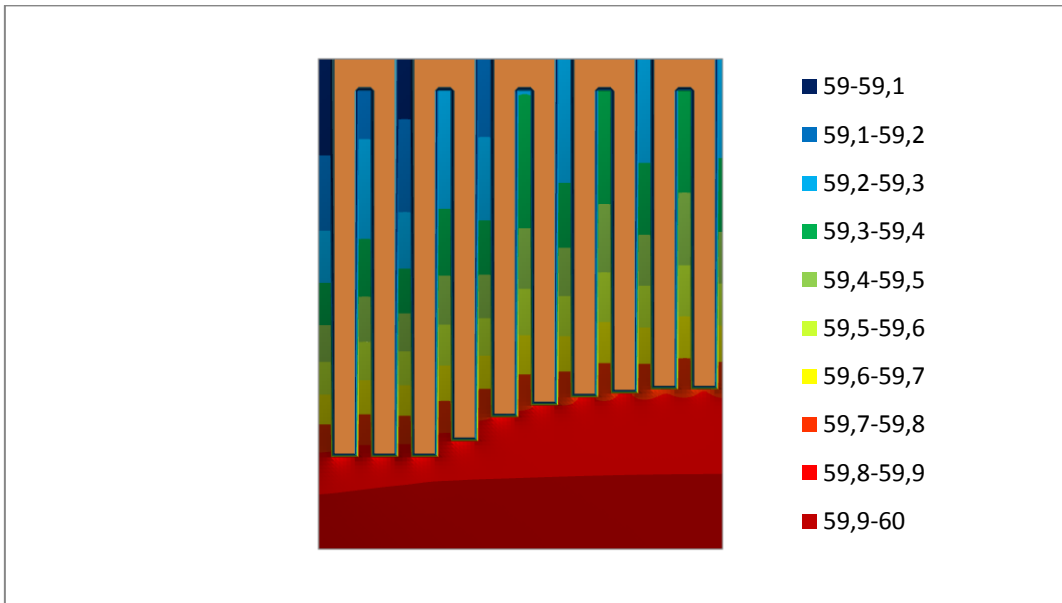
## Aluminum results

Table 5. Aluminum, natural convection. Results

n	Qd	Qw	Qd					Qw				
			%	r	r (%)	Q	Q (%)	%	r	r (%)	Q	Q (%)
2000	27,10462	117,31278										
4000	48,93465	117,00981										
6000	65,03857	116,77932										
8000	77,18703	116,60356		0,7604	2,4688				0,7641	2,0868		
10000	86,40526	116,46939	25,5321	0,7613	2,3496	115,76841		0,3785	0,7648	2,0044	116,0336	
12000	93,41447	116,36686	19,4913	0,7621	2,2514	115,84027	0,1637005	0,2901	0,7654	1,9262	116,0327	0,002124
14000	98,75052	116,28846	14,8924	0,7628	2,1614	115,89082	0,1201372	0,2225	0,7659	1,8516	116,0321	0,001575
16000	102,81690	116,22845	11,3878	0,7634	2,0764	115,92744	0,0885728	0,1708	0,7665	1,7801	116,0316	0,001171
18000	105,91856	116,18248	8,7147	0,7641	1,9955	115,95422	0,0654919	0,1312	0,7670	1,7115	116,0312	0,000872
20000	108,28645	116,14725	6,6739	0,7647	1,918	115,97389	0,0485424	0,1008	0,7676	1,6455	116,0310	0,000650
22000	110,09563	116,12023	5,1147	0,7652	1,8438	115,98838	0,0360575	0,0776	0,7681	1,5820	116,0308	0,000486
24000	111,47904	116,09948	3,9224	0,7658	1,7725	115,99907	0,026837	0,0597	0,7685	1,5207	116,0307	0,000363
26000	112,53767	116,08355	3,0100	0,7663	1,704	116,00699	0,0200105	0,0460	0,7690	1,4615	116,0306	0,000272
28000	113,34836	116,07131	2,3113	0,7668	1,6378	116,01287	0,0149449	0,0354	0,7694	1,4042	116,0305	0,000204
30000	113,96962	116,06189	1,7759	0,7673	1,574	116,01724	0,0111779	0,0273	0,7699	1,3488	116,0304	0,000153
32000	114,44602	116,05465	1,3653	0,7678	1,5123	116,0205	0,0083712	0,0210	0,7703	1,2951	116,0304	0,000115
34000	114,81158	116,04907	1,0503	0,7683	1,4525	116,02293	0,0062761	0,0162	0,7707	1,2430	116,0303	0,000087
36000	115,09227	116,04478	0,8084	0,7687	1,3946	116,02475	0,0047096	0,0125	0,7711	1,1924	116,0303	0,000065
38000	115,30792	116,04146	0,6225	0,7692	1,3384	116,02611	0,0035366	0,0097	0,7715	1,1433	116,0303	0,000049
40000	115,47369	116,03891	0,4796	0,7696	1,2838	116,02713	0,0026571	0,0075	0,7718	1,0956	116,0303	0,000037
42000	115,60120	116,03694	0,3697	0,7700	1,2307	116,0279	0,0019967	0,0058	0,7722	1,0492	116,0303	0,000028
44000	115,69934	116,03542	0,2852	0,7704	1,1791	116,02847	0,0015004	0,0045	0,7726	1,0040	116,0303	0,000021
46000	115,77490	116,03425	0,2200	0,7708	1,1288	116,02891	0,001127	0,0035	0,7729	0,9602	116,0303	0,000016
48000	115,83312	116,03334	0,1699	0,7712	1,0799	116,02923	0,0008459	0,0027	0,7732	0,9175	116,0302	0,000012
50000	115,87799	116,03264	0,1312	0,7716	1,0323	116,02948	0,000634	0,0021	0,7736	0,8759	116,0302	0,000009
52000	115,91259	116,03210	0,1014	0,7719	0,9859	116,02966	0,0004742	0,0016	0,7739	0,8355	116,0302	0,000007
54000	115,93930	116,03168	0,0784	0,7723	0,9407	116,0298	0,0003536	0,0012	0,7742	0,7961	116,0302	0,000005
56000	115,95991	116,03135	0,0606	0,7726	0,8966	116,02991	0,0002626	0,0010	0,7745	0,7579	116,0302	0,000004
58000	115,97582	116,03110	0,0469	0,7730	0,8537	116,02999	0,0001939	0,0007	0,7748	0,7206	116,0302	0,000003
60000	115,98812	116,03091	0,0363	0,7733	0,8118	116,03005	0,000142	0,0006	0,7751	0,6843	116,0302	0,000002
118000	116,0302068	116,0302358										
120000	116,030213	116,0302357										

$$Q_{Al\ natural} = 116,0302 \frac{W}{m}$$

Figure 20. Aluminum, natural convection. Temperature distribution graphic



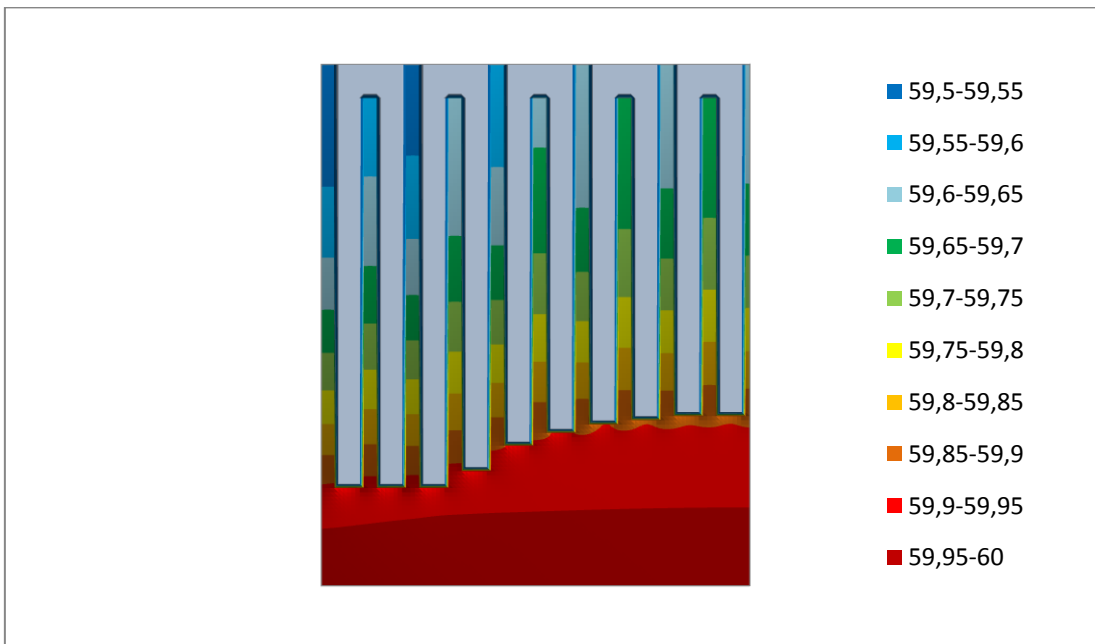
## Cooper Results

Table 6. Cooper, natural convection. Results

n	Qd	Qw	Qd					Qw				
			%	r	r (%)	Q	Q (%)	%	r	r (%)	Q	Q (%)
2000	27,108293	117,513691										
4000	48,990845	117,359774										
6000	65,166691	117,242432										
8000	77,395505	117,152759		0,7620	2,5566				0,7658	2,160		
10000	86,694731	117,084155	25,8116	0,7629	2,4373	116,5860		0,1940	0,7664	2,077	116,8593	
12000	93,780795	117,031620	19,7477	0,7637	2,3392	116,6600	0,1689826	0,1490	0,7670	1,999	116,8588	0,001112
14000	99,187026	116,991357	15,1214	0,7644	2,2492	116,7122	0,124313	0,1145	0,7676	1,925	116,8585	0,000827
16000	103,315796	116,960475	11,5882	0,7651	2,1642	116,7501	0,0918717	0,0881	0,7682	1,853	116,8582	0,000616
18000	106,471867	116,936770	8,8874	0,7657	2,0832	116,7779	0,0680943	0,0678	0,7687	1,784	116,8580	0,000460
20000	108,886496	116,918560	6,8211	0,7663	2,0058	116,7984	0,0505928	0,0522	0,7692	1,718	116,8579	0,000344
22000	110,735396	116,904562	5,2389	0,7669	1,9315	116,8135	0,0376716	0,0403	0,7697	1,655	116,8578	0,000257
24000	112,152234	116,893795	4,0265	0,7675	1,8602	116,8247	0,0281067	0,0311	0,7702	1,593	116,8577	0,000193
26000	113,238797	116,885507	3,0967	0,7680	1,7916	116,8329	0,021009	0,0240	0,7707	1,534	116,8577	0,000145
28000	114,072683	116,879124	2,3831	0,7685	1,7254	116,8391	0,0157301	0,0185	0,7711	1,477	116,8576	0,000109
30000	114,713096	116,874205	1,8351	0,7690	1,6616	116,8437	0,0117956	0,0143	0,7715	1,421	116,8576	0,000082
32000	115,205258	116,870412	1,4139	0,7695	1,5998	116,8471	0,0088574	0,0110	0,7720	1,368	116,8576	0,000062
34000	115,583733	116,867485	1,0900	0,7700	1,5400	116,8497	0,0066592	0,0085	0,7724	1,315	116,8576	0,000047
36000	115,874965	116,865226	0,8408	0,7704	1,4820	116,8516	0,0050119	0,0066	0,7728	1,265	116,8575	0,000035
38000	116,099201	116,863481	0,6489	0,7708	1,4257	116,8531	0,0037756	0,0051	0,7732	1,216	116,8575	0,000027
40000	116,271954	116,862132	0,5011	0,7713	1,3711	116,8542	0,0028465	0,0040	0,7735	1,168	116,8575	0,000020
42000	116,405121	116,861090	0,3871	0,7717	1,3180	116,8550	0,0021473	0,0031	0,7739	1,121	116,8575	0,000015
44000	116,507829	116,860283	0,2992	0,7721	1,2663	116,8556	0,0016207	0,0024	0,7742	1,076	116,8575	0,000011
46000	116,587088	116,859659	0,2314	0,7725	1,2160	116,8561	0,0012236	0,0018	0,7746	1,032	116,8575	0,000009
48000	116,648284	116,859176	0,1790	0,7729	1,1671	116,8564	0,000924	0,0014	0,7749	0,989	116,8575	0,000007
50000	116,695557	116,858802	0,1386	0,7732	1,1194	116,8567	0,0006976	0,0011	0,7752	0,948	116,8575	0,000005
52000	116,732093	116,858512	0,1073	0,7736	1,0730	116,8569	0,0005266	0,0009	0,7756	0,907	116,8575	0,000004
54000	116,760344	116,858287	0,0831	0,7740	1,0277	116,8570	0,0003972	0,0007	0,7759	0,868	116,8575	0,000003
56000	116,782200	116,858112	0,0644	0,7743	0,9836	116,8571	0,0002994	0,0005	0,7762	0,830	116,8575	0,000002
58000	116,799115	116,857977	0,0500	0,7746	0,9406	116,8572	0,0002253	0,0004	0,7765	0,792	116,8575	0,000002
60000	116,812213	116,857872	0,0387523	0,774970618	0,8987	116,8573	0,0001692	0,0003	0,7767	0,756	116,8575	0,000001
128000	116,8574956	116,8575053										
130000	116,8574977	116,8575053										

$$Q_{Cu natural} = 116,8575 \frac{W}{m}$$

Figure 21. Cooper, natural convection. Temperature Distribution graphic



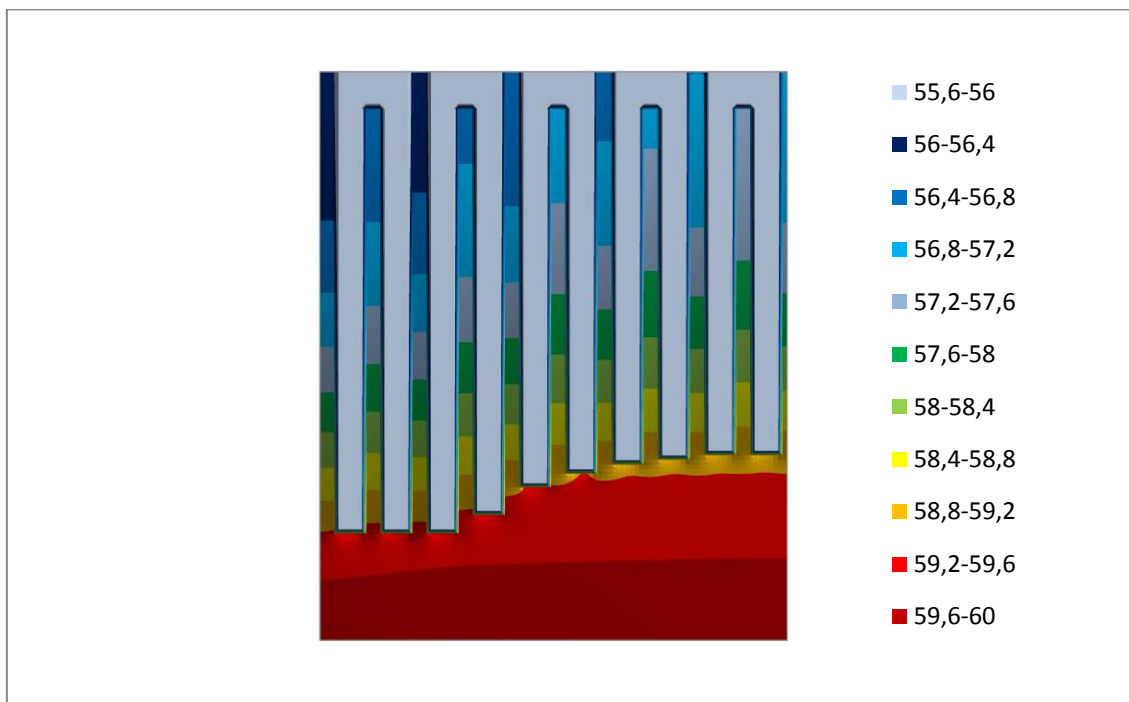
## Steel Results

Table 7. Steel, natural convection. Results

n	Qd	Qw	Qd					Qw				
			%	r	r (%)	Q	Q (%)	%	r	r (%)	Q	Q (%)
2000	26,979244	115,923771										
4000	48,473318	114,606589										
6000	64,102485	113,619493										
8000	75,718044	112,878016		0,7490	2,5225				0,7527	2,1305		
10000	84,400373	112,320409	23,6938	0,7499	2,4041	110,4030		1,5487	0,7533	2,0487	110,6189	
12000	90,903417	111,900695	17,8144	0,7507	2,3065	110,4616	0,131852	1,1692	0,7539	1,9712	110,6157	0,007508
14000	95,780102	111,584510	13,4054	0,7513	2,2171	110,5022	0,095172	0,8834	0,7545	1,8972	110,6135	0,005479
16000	99,440823	111,346129	10,0957	0,7520	2,1327	110,5311	0,069024	0,6678	0,7550	1,8264	110,6119	0,004006
18000	102,191287	111,166269	7,6090	0,7526	2,0524	110,5519	0,050212	0,5052	0,7556	1,7585	110,6107	0,002936
20000	104,259616	111,030468	5,7391	0,7532	1,9756	110,5669	0,036619	0,3825	0,7561	1,6933	110,6098	0,002155
22000	105,816263	110,927861	4,3317	0,7538	1,9021	110,5778	0,026767	0,2897	0,7566	1,6305	110,6092	0,001585
24000	106,988729	110,850282	3,2717	0,7543	1,8315	110,5858	0,019606	0,2195	0,7570	1,5699	110,6087	0,001167
26000	107,872493	110,791591	2,4727	0,7548	1,7637	110,5915	0,014389	0,1665	0,7575	1,5115	110,6084	0,000861
28000	108,539122	110,747160	1,8700	0,7553	1,6983	110,5957	0,010579	0,1263	0,7579	1,4550	110,6081	0,000636
30000	109,042313	110,713505	1,4150	0,7558	1,6352	110,5988	0,007790	0,0959	0,7583	1,4003	110,6080	0,000470
32000	109,422389	110,687998	1,0714	0,7563	1,5742	110,6011	0,005745	0,0728	0,7587	1,3474	110,6078	0,000348
34000	109,709656	110,668656	0,8117	0,7567	1,5152	110,6027	0,004243	0,0553	0,7591	1,2960	110,6077	0,000257
36000	109,926912	110,653980	0,6153	0,7572	1,4580	110,6040	0,003137	0,0421	0,7595	1,2462	110,6077	0,000191
38000	110,091318	110,642839	0,4666	0,7576	1,4025	110,6049	0,002321	0,0320	0,7599	1,1978	110,6076	0,000142
40000	110,215802	110,634378	0,3541	0,7580	1,3486	110,6055	0,001720	0,0244	0,7602	1,1508	110,6076	0,000105
42000	110,310112	110,627948	0,2688	0,7584	1,2963	110,6060	0,001275	0,0185	0,7606	1,1051	110,6075	0,000078
44000	110,381600	110,623060	0,2042	0,7588	1,2453	110,6064	0,000946	0,0141	0,7609	1,0607	110,6075	0,000058
46000	110,435818	110,619342	0,1552	0,7592	1,1958	110,6067	0,000702	0,0108	0,7613	1,0174	110,6075	0,000043
48000	110,476960	110,616513	0,1180	0,7596	1,1475	110,6069	0,000521	0,0082	0,7616	0,9754	110,6075	0,000032
50000	110,508194	110,614360	0,0897	0,7599	1,1006	110,6070	0,000387	0,0063	0,7619	0,9345	110,6075	0,000024
52000	110,531918	110,612720	0,0683	0,7603	1,0548	110,6071	0,000287	0,0048	0,7622	0,8947	110,6075	0,000018
54000	110,549947	110,611470	0,0520	0,7606	1,0102	110,6072	0,000213	0,0036	0,7625	0,8559	110,6075	0,000013
56000	110,563654	110,610517	0,0396	0,7610	0,9667	110,6073	0,000158	0,0028	0,7628	0,8182	110,6075	0,000010
58000	110,574080	110,609791	0,0302	0,7613	0,9244	110,6073	0,000117	0,0021	0,7631	0,7815	110,6075	0,000007
128000	110,60744	110,6074447										
130000	110,60744	110,6074447										

$$Q_{Steel\ natural} = 110,6074 \frac{W}{m}$$

Figure 22. Steel, natural convection. Temperature distribution graphic



The main point after evaluating the last tables is the decreasing of heat dissipation while heat is transferred to the environment with natural convection; the convection coefficient is 16,6 times lower. Moreover, as it will be shown in the next paragraphs the differences between the natural convection values are not as big as the ones that took place using forced convection.

### 3.2.3 Comparison between forced and natural convection

The table below presents the main factors evaluated during the accomplishment of the tests. The aim is compare the range of heat dissipated in every case and try to deliver a view about the usage of the method by specifying the iterations needed to find results enough approached to the real values of this heat transfer.

Table 8. Comparison between forced and natural convection

	Forced Convection $\alpha = 100 \text{ W/m}^2\text{K}$			Natural Convection $\alpha = 6 \text{ W/m}^2\text{K}$		
	Cu	Al	Steel	Cu	Al	Steel
Heat dissipated (W/m)	1749,90769	1587,4862	999,6923	116,8575	116,030213	110,6074
Lower Temperature (°C)	52,99958551	47,9080	31,7195	59,5132	59,04812297	56,0392
Iterations to get 0,0001% mistake in the prediction	Qd	50000	58000	22000	60000	60000
	Qw	38000	48000	20000	24000	30000
% Heat Dissipation comparing with Aluminum	110,2313639	100,0000	62,97329073	100,7130	100	95,3264

As the table shows, the usage of natural convection affects gradually the heat dissipated by an enormous decreasing. The importance of the sort of convection is clearer when we noticed the lower difference between the heat transferred by each material. The maximum difference with the aluminum (the reference) is, in this case, less than 5% while in forced convection is around 37%. Also with the temperatures, the small value of the convection coefficient does not allow the cooling of the piece with the environment (20°C).

The conclusion is that the best material in order to build the finned surface would be cooper, but only attending to thermo dynamical aspects.



### 3.3 Heat flux from the base and Isolation. Second Kind of Boundary Condition.

Using the reference material (Aluminum) another sort of test has been made. The aim was the checking MS EXCEL usage changing also the boundary conditions in the base of the body.

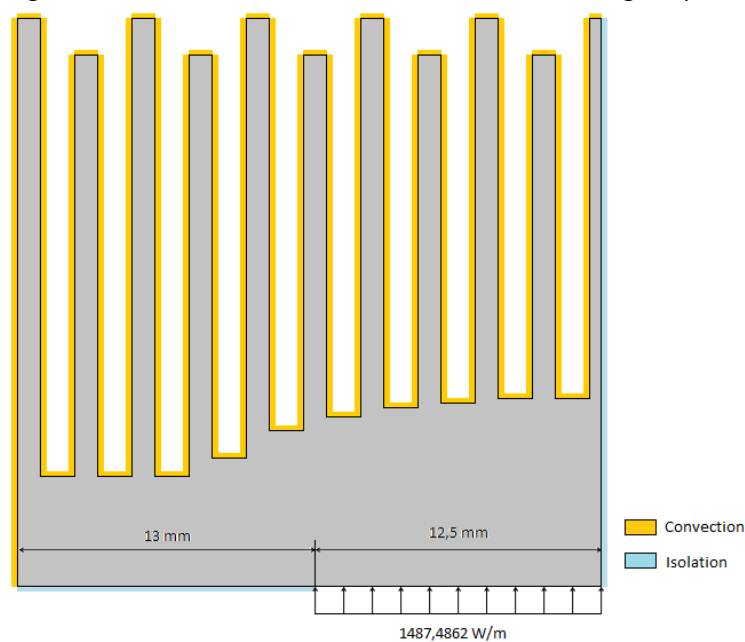
Once the value of heat transferred was achieved in the first aluminum test, and also in forced convection conditions, the idea was introduce the same amount of heat across the base surface with the objective of dissipate it using the fins. Therefore, along the base we find the Second Kind of Boundary Conditions:

$$\dot{Q}_{Bi} = \dot{q} \cdot \Delta x \cdot 1 \quad (3.9)$$

Where  $\dot{Q}_{Bi}$  is the heat transferred between the outside and one single cell, and  $\dot{q}$  is the heat flux given.

In order to performance the new case, one part of the base piece was isolated and the rest of it was designed to transfer the 1487,4862 W/m. The isolated portion of the base is 13 mm long and the heat flux is transferred along the 12,5 mm left. This last length corresponds with 50 single cells in MS EXCEL.

Figure 23. Convection, isolation and heat flux surrounding the piece





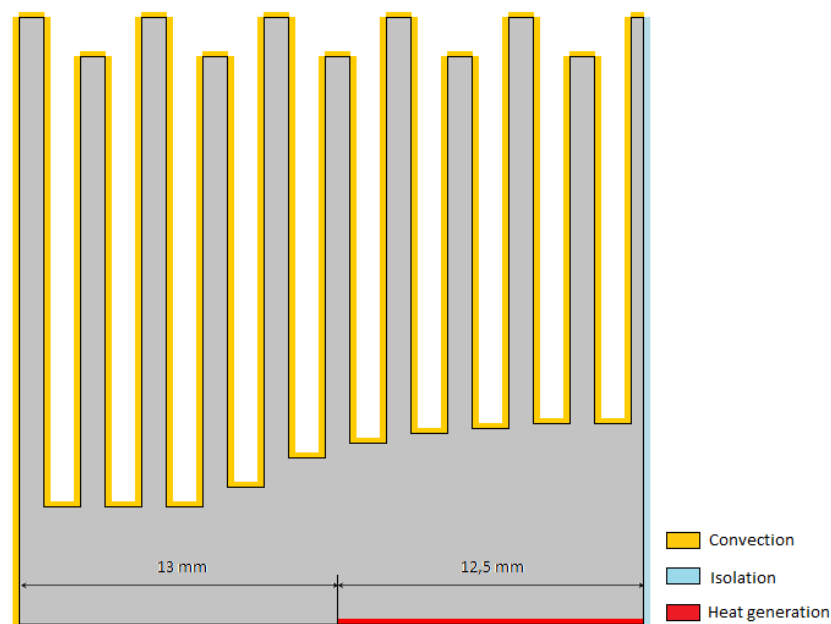
### 3.3.1 Heat: Isolation & Generation

The performance of the part in where is needed the heat flux begins with the specification of the total amount of heat needed, as is been explained, corresponds to 1487,4862 W/m shared for 50 single cells. That means that every cell must transfer from the base 29,75 W/m approximately.

After several ways tried to prepare the solution, the final idea was using the 50 cells belonging to the piece base to *generate* the heat flux needed. Thus, some changes had to be made in the equations and, obviously, by using *generation* in the inside cells, the base was considered to be isolated from the environment.

In other words, the best performance was the changing of *heat flux from the outside* for the *heat generation inside*, the problem to solve remains the same. The next graphic shows the final draft used to find the solution:

Figure 24. Convection, isolation and heat generation



In all the previous tests the term of the equation (3.2) corresponding to the heat generation was zero. In this case, it has the value required by every cell which generates heat. That term corresponds to the 29,75 W/m that every cells has to generate. Also, if it is considered the whole base isolated, the conductance  $k_D$  must be zero along the base.

In the next paragraphs are presented the results found for this second case. May notice that the main difference between the previous ones lie in the temperature distribution graphic.

### 3.3.2 Results

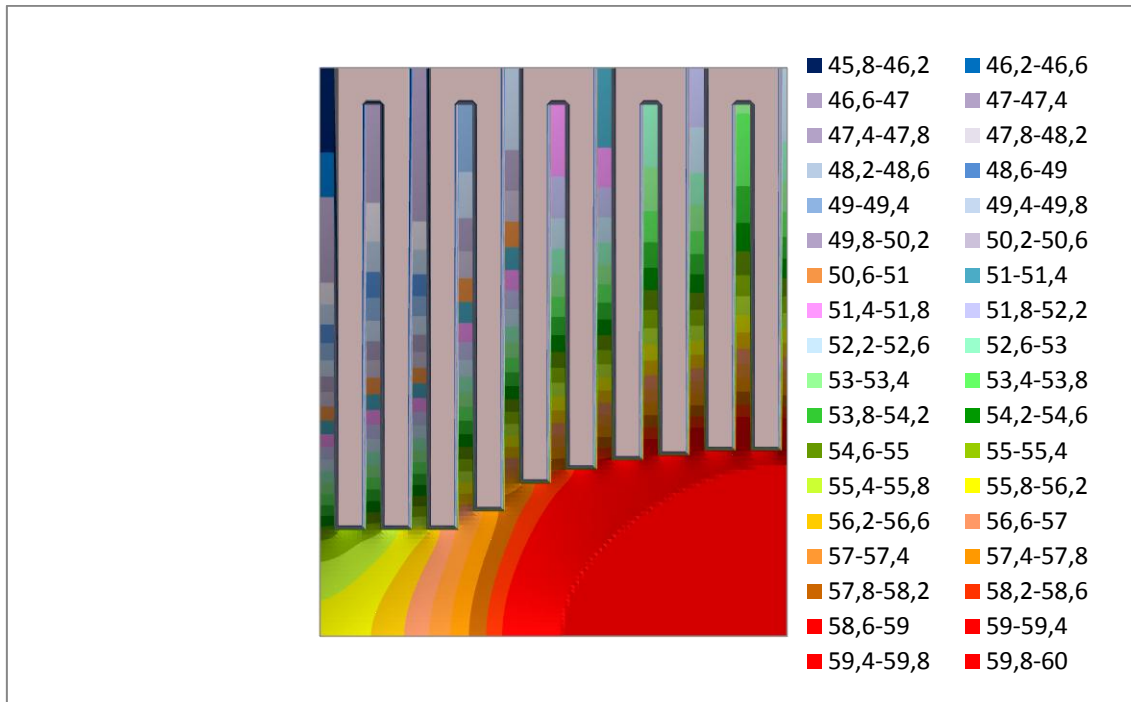
Table 9. Aluminum, second case, forced convection. Results

n	Qw	Qw				
		%	r	r (%)	Q	Q (%)
2000	1655,099528					
4000	1623,528531					
6000	1606,022109					
8000	1596,110459		0,5872	39,126694		
10000	1590,390603	-0,1830	0,5954	38,284446	1582,0900	
12000	1587,031756	0,0286	0,5993	37,873818	1582,0411	0,3430010
14000	1585,032063	0,1546	0,5958	38,242955	1582,0675	0,3413413
16000	1583,833623	0,2301	0,5790	39,977617	1582,1377	0,3369190
18000	1583,119651	0,2751	0,5377	44,264612	1582,2255	0,3313859
20000	1582,706249	0,3011	0,4411	54,274184	1582,3086	0,3261538
22000	1582,483978	0,3151	0,1620	83,211066	1582,3670	0,3224730
24000	1582,385934	0,3213	-2,4627	355,29452	1582,3813	0,3215682
26000	1582,370055	0,3223	1,9452	-101,64931	1582,3287	0,3248857
28000	1582,409161	0,3198	1,3253	-37,382339	1582,1753	0,3345477
30000	1582,485232	0,3150	1,1617	-20,423054	1581,8617	0,3543025
32000	1582,586048	0,3087	1,0886	-12,848617	1581,2644	0,3919258
34000	1582,703163	0,3013	1,0485	-8,6879848	1580,0733	0,4669584
36000	1582,830657	0,2933	1,0238	-6,1328921	1577,2214	0,6466065
38000	1582,964332	0,2848	1,0076	-4,4518328	1564,9907	1,4170520
40000	1583,101193	0,2762	0,9964	-3,293876	1621,8822	-2,1666979
42000	1583,239096	0,2675	0,9885	-2,4703094	1595,1873	-0,4851159
44000	1583,376508	0,2589	0,9827	-1,8709485	1591,2360	-0,2362120
46000	1583,512340	0,2503	0,9784	-1,4273398	1589,7031	-0,1396497
48000	1583,645825	0,2419	0,9752	-1,0948615	1588,9188	-0,0902414
50000	1583,776431	0,2337	0,9728	-0,8433067	1588,4599	-0,0613373
52000	1583,903802	0,2257	0,9710	-0,6516065	1588,1698	-0,0430641
54000	1584,027710	0,2179	0,9695	-0,5047129	1587,9772	-0,0309265
150000	1586,880306					
400000	1587,482757					
	1587,4862					

As is presented in the table above, the iterations needed to approach the heat convection to the heat crossing the piece base are many more than in the last examples, 400000 iterations. From the iteration 54000 is possible to predict the converging value with a 0,03% of inaccuracy, taking into account the big number of iterations needed to converge, the method offers a good way to approach the final result.

The reference value is 1587,4862 W/m calculated in the previous tests for the aluminum with forced convection

Figure 25. Aluminum, second case, forced convection. Temperature distribution graphic



In the graphic above is easy to identify the area where the heat flux is given. The isotherms in this case change, they draw leaned shapes and the maximum temperatures are found where the heat source is.

### References

- [5] Yunus A. Çengel, *"Heat Transfer. A practical Approach"*, 1998 McGraw Hill

## 4 Heat transfer from finned surfaces

### 4.1 Theory Introduction

The aim of the chapter number 4 is present the techniques for the final calculation of the heat transfer in the fin effectiveness and the proper value of the dimensions that the piece should have to dissipate as much heat as possible.

The purpose of the fins in any body is to improve the heat dissipation from the surface. They often increase the heat transfer due to the bigger area in contact with the surrounding fluid:

$$\dot{Q} = \alpha A(T_s - T_\infty) \quad \text{if } A \uparrow \rightarrow Q \uparrow \quad (4.1)$$

However, it is not always certain that if the  $A$  is increased the heat flux is consequently enhanced. The convection equation (4.1) is also governed by the value of the convection coefficient ( $\alpha$ ). If  $\alpha$  is not considered constant is necessary to remark that its value can change from the fin base until the fin tip. The value of  $\alpha$  is usually much lower in the fin base because the fluid is surrounded by solid surfaces near the bottom of the fin, which seriously disrupts its motion to the point of “suffocating”, while the movements of the solid in the fin tip are almost free due to the little resistance the fluid find around the tip, increasing the value of  $\alpha$ . Thus, the fact of disposing a large number of fins does not guarantee totally a better heat transfer.

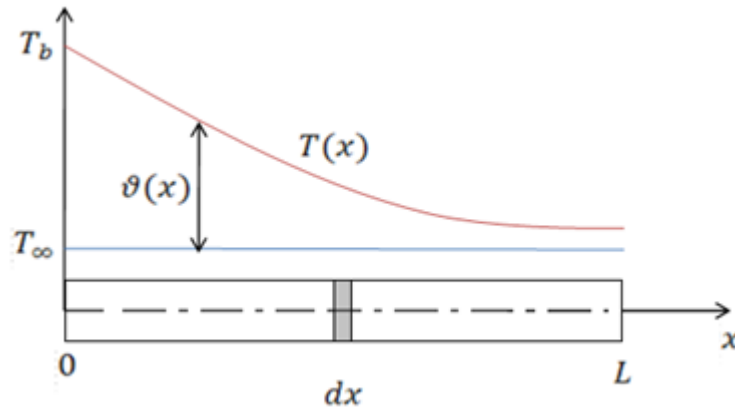
In the current analysis both the value of  $\alpha$  (convection coefficient) and  $\lambda$  (thermal conductivity along the body) are considered constant and only depend on the surrounding fluid plus the sort of convection and the piece material respectively. As it has been shown in previous chapters, different tests have been made by changing the kind of convection (natural or forced) or the material of the studied body (Al, Cu and Steel), but always considering equal the value of  $\lambda$  among the body and the value of  $\alpha$  around it.

In order to calculate the heat transfer through the fin could be used two different methods. The first one is based in the heat conduction among the fin and the second one in the convection with the surrounding fluid. Both of them offers the same result due to the fact that the heat crossing the fin by conduction must be equal to the heat transferred to the environment or dissipated, under steady conditions.

## Conduction

This analytic expression comes from the Fourier's Law and is not anything else than the heat conduction transfer crossing a given section of the fin.

Figure 26. Heat dissipation graphic along fin



$$\dot{Q}_{fin} = \lambda \cdot A_c \cdot \left. \frac{\partial \vartheta}{\partial x} \right|_{x=0} \quad (4.2)$$

where

$A_c = A_{base}$  : The normal area to heat flux

$$\text{and} \quad \vartheta(x) = T(x) - T_{\infty} \quad (4.3)$$

This expression is developed depending on the considerations taken for solving every kind of problem. Afterwards is shown the 3 main cases for fit the expression (4.2) to a specific problem.

## Convection

The convection expression is based in the dissipation that takes place in every fin side in contact with the environment. It is necessary to take into account that the fin temperature will be different from the base to the tip, so that it will be logical to divide the fin in  $x$  differentials in order to be as much accurate as possible in the results. Then the sum of the dissipation in every  $dx$  is carried out with an integral.

Therefore, the equation depends on the convection constant, the fin geometry parameters and the given environmental temperature:

$$\dot{Q}_{fin} = \int_0^L d\dot{Q}_0 = \int_0^L \alpha \cdot p \cdot (T(x) - T_{\infty})dx = p \cdot \alpha \int_0^L \vartheta(x) \cdot dx \quad (4.4)$$

The expression above is under the next considerations:  $\alpha$  constant along the length of the fin and constant value of the perimeter ( $p$ ) due to the considered uniform cross section.

The fin tip offers several possibilities. In the next table are summarized each case separately with their respective expressions.

Table 10. Heat expressions depending on fins descriptions and boundary condition

Description	Boundary condition	Expressions
<b>Infinitely long fin</b> The temperature of the sufficient long fin will approach the environment temperature.	$\vartheta(L) = T(L) - T_{\infty}$ $\vartheta(L) = 0$ $L \rightarrow \infty$	$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = e^{-a \cdot x}$ $a = \sqrt{\frac{\alpha \cdot p}{\lambda \cdot A_c}}$ $\dot{Q}_{fin} = \sqrt{\alpha p \lambda A_c} (T_b - T_{\infty})$
<b>Insulated fin tip</b> More realistic situation. Fins are not used to be that long, so that is a good solution consider the heat flux of the fin tip negligible in comparison with total heat transfer.	$\left. \frac{d\vartheta}{dx} \right _{x=L} = 0$	$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh a(L - x)}{\cosh aL}$ $\dot{Q}_{fin} = \sqrt{\alpha p \lambda A_c} (T_b - T_{\infty}) \cdot \tanh aL$
<b>Convection</b> More accuracy. In practice, the fin tips are exposed to the surroundings. A practical way of accounting it, is to replace the fin length in the relation for the insulated tip case by a corrected length called $L_c$ .	$\left. \frac{d\vartheta}{dx} \right _{x=L_c} = 0$ $L_c = L + \frac{A_c}{p}$	$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh a(L - x)}{\cosh aL}$ $\dot{Q}_{fin} = \sqrt{\alpha p \lambda A_c} (T_b - T_{\infty}) \cdot \tanh aL$

where  $p$  is the perimeter,  $A_c$  the cross section,  $\alpha$  the convection coefficient and  $L$  the length of the fin.

## 4.2 Efficiency & Effectiveness of the Fins

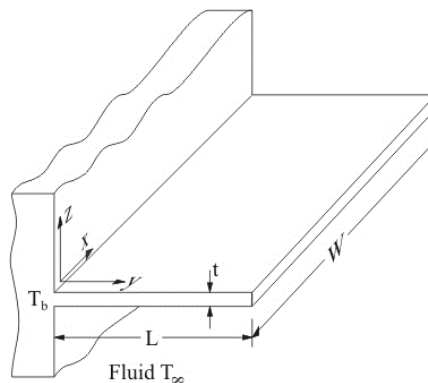
In order to evaluate the usage of the fins in this work, it has been calculated both efficiency and effectiveness of them. As it will be shown in the next paragraphs, the geometry of the studied piece has required a specific methodology to achieve coherent results. However, all the calculations proceed from the pattern learnt in the general equations.

### 4.2.1 Fin efficiency

Fin efficiency is defined as the relation between the actual heat transfer from the fins surface and the maximum heat possible to transfer across the same area. The difference between this two concepts lies in the temperature distribution applied for calculate it.

Without focusing in the studied body, the efficiency of a regular fin with the geometry shown in the graphic below would be calculated using the real area constituted by the fin surface.

Figure 27. Fin example



$$A_{fin} = 2 \cdot w \cdot L + w \cdot t \cong 2 \cdot w \cdot L \quad (4.5)$$

It is considered a fin of a constant cross-sectional area  $A_c = A_b$ , like in the studied piece. The temperature of the fin will be  $T_b$  at the fin base and gradually decrease towards the fin tip. Thus, the heat dissipated by convection should be less due to a lower temperature difference between the fin and the environment. However, the surface is increased with the fin. Consequently, the efficiency value will deliver the worthiness of using an extended surface taking into account the decrease of the temperature along the fin length. In the limiting case of *zero thermal resistance* or *infinite thermal conductivity* ( $\lambda \rightarrow \infty$ ), the



temperature of the fin will be uniform and equal to the base value of  $T_b$ . The heat transfer from the fin will be *maximum* in this case and can be expressed as:

$$\dot{Q}_{fin,max} = \alpha \cdot A_{fin} \cdot (T_b - T_{\infty}) \quad (4.6)$$

Then, the efficiency expression can be written as:

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{finmax}} \quad (4.7)$$

where

$\dot{Q}_{fin}$  : Actual heat transfer.

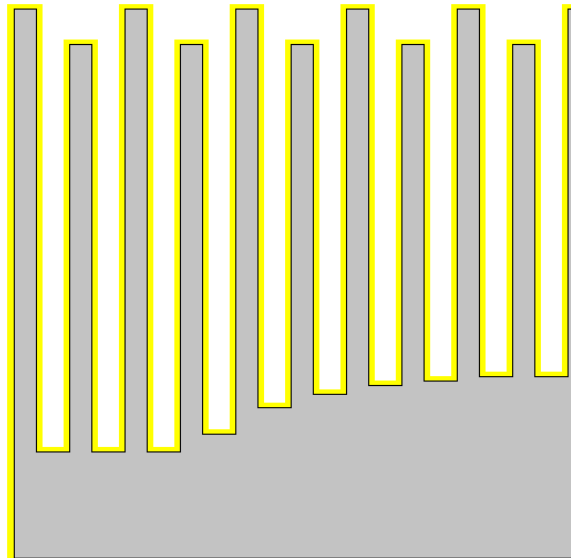
$\dot{Q}_{finmax}$  : Ideal heat transfer if the entire fin were at base temperature.

Once the efficiency of a specific fin is known, is easy to calculate the actual heat transfer in this body from the fins:

$$\dot{Q}_{fin} = \eta_{fin} \cdot \alpha \cdot A_{fin} \cdot (T_b - T_{\infty}) \quad (4.8)$$

In our case a new efficiency concept has had to be introduced in order to compare the heat transferred from the body and the maximum heat if the whole piece would keep the base temperature. The piece is not fully constituted by fins, but it has space between the base and the beginning of all the fins. Furthermore, this space is not regular along the body. Another question is the importance of evaluate the comparison using the total heat transfer from the body, using both extended surface and the *unfin surface* (area between fins). In this case, we will use the calculated heat transfer with the iterations, the total amount. So that, we will analyze the whole body efficiency, how much it is dissipated related with the maximum value. In the next graphic it is shown the surrounding area used for that aim.

Figure 28. Area used to calculate  $\dot{Q}_{fin}$



The surface painted in yellow represents the total heat transfer of the piece; we may notice that the area between the fins also is useful to determine the total heat transfer. If we are coherent with this idea, it is necessary to express the maximum heat transfer formula using the same area  $A_{total}$ .

$$\dot{Q}_{max} = \alpha \cdot A_{total} \cdot (T_b - T_{\infty}) \quad (4.9)$$

where  $T_b = 60^{\circ}\text{C}$  and  $T_{\infty} = 20^{\circ}\text{C}$

Consequently, we define the heat relation as a *fin surface efficiency*, whose formula looks similar to (4.7) but its concept is slightly modified:

$$\eta_{fin\ surface} = \frac{\dot{Q}_{total}}{\dot{Q}_{max}} \quad (4.10)$$

where

$\dot{Q}_{total}$  : Actual total amount of heat transferred by convection

$\dot{Q}_{max}$  : Dissipation considering the whole body at base temperature

Usually the determination of any area belonging to any piece is carried out with specific formulas depending on the type of fins. MS EXCEL allows calculating it measuring it manually.

#### 4.2.2 Fin effectiveness

As it is been explained in last paragraphs the usage of the fins do not assure an enhancement of heat dissipation. In order to check the benefit of the disposal of the fins it is needed other factors. The improvement of the heat dissipation should also explain the added cost and complexity associated with the fins and its geometry. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case. Therefore, the next factor to take into account, *fin effectiveness*  $\varepsilon_{fin}$ , should compare the heat dissipation from the fins surface and the heat dissipation from the cross-sectional area of the fins at the base ( $A_{base}$ ) if no fins are attached in the surface. Thus, for a regular fin surface, the expression would be:

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{\alpha \cdot A_{base} \cdot (T_b - T_{\infty})} \quad (4.11)$$

The three main sorts of results we can come across are:

$\varepsilon_{fin} < 1$ : Indicates that the fin actually act as *insulation*, slowing down the heat transfer from the surface

$\varepsilon_{fin} = 1$ : Indicates that the addition of fins does not affect to the heat transfer at all. Therefore, the disposal of the fins does not worth due to it added cost.

$\varepsilon_{fin} > 1$ : Indicates that the fins are enhancing heat transfer from the surface, as they should.

In a regular fin performance is possible to turn the expression (4.11) into something more specific. The next equation is not proper to our study because of the differences introduced in the point 4.2.1.

$$\begin{aligned} \varepsilon_{fin} &= \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{h \cdot A_{base} \cdot (T_b - T_{\infty})} = \\ &= \frac{\eta_{fin} \cdot \alpha \cdot A_{fin} \cdot (T_b - T_{\infty})}{\alpha \cdot A_{base} \cdot (T_b - T_{\infty})} = \frac{A_{fin}}{A_{base}} \cdot \eta_{fin} \end{aligned} \quad (4.12)$$

The differences between all the fins in our study do not allow a useful way to calculate the *fin effectiveness*, moreover the determination of this value is useless for next calculation and does not show any added information. Furthermore, the temperature between the fin

base and the environment is not clear due to the fact that the temperature rate decreases from the base of the piece and the base of the fins. Therefore, the effort to calculate  $\dot{Q}_{no\ fin}$  does not worth.

### 4.2.3 Overall fin effectiveness

The *overall fin effectiveness* for a finned surface is the ratio of the total heat dissipated from the finned surface to the heat transfer from the same surface if there were no fins.

This relation gives an accurate idea of the usage of the fins. The total heat dissipated is calculated considering also the *unfinned portion* of the surface or the surface between fins as well as the fins. This value has been used also in the point 4.2.1 for calculate *fin surface efficiency*.

In a regular fin, where we do not dispose of its real geometry, the way to determine the total heat may be expressed as:

$$\dot{Q}_{total} = \dot{Q}_{unfin} + \dot{Q}_{fin} = \alpha \cdot (A_{unfin} + \eta_{fin} A_{fin})(T_b - T_{\infty}) \quad (4.13)$$

Thus, once the *efficiency* has been calculated, the total heat transfer is achieved.

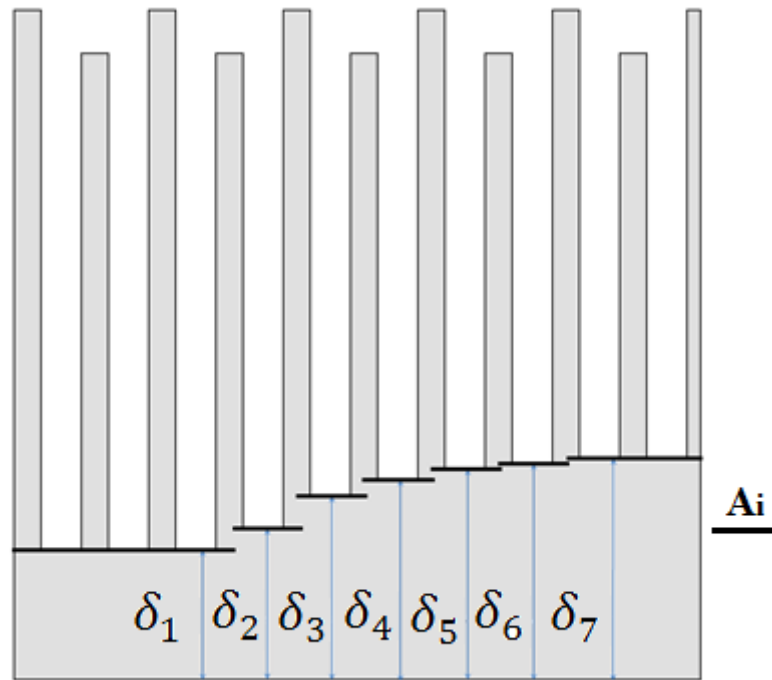
About the *overall fin effectiveness*:

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,fin}}{\dot{Q}_{total,nofin}} = \frac{\alpha \cdot (A_{unfin} + \eta_{fin} A_{fin})(T_b - T_{\infty})}{\alpha \cdot A_{no\ fin}(T_b - T_{\infty})} \quad (4.14)$$

Where  $A_{no\ fin}$  is the area of the surface when there are no fins,  $A_{fin}$  is the total surface area of all the fins on the surface, and  $A_{unfin}$  is the area of the unfinned portion of the surface.

In our project is not necessary to make part of the last determinations due to the disposal of the computer performance of our piece. The  $\dot{Q}_{total,fin}$  is already calculated by the program. About  $\dot{Q}_{total,nofin}$ , the accomplishment of its value has not been as easy as the formula (4.14) shows. The heat dissipation is not uniform in the entire surface without fins, so that, we cannot express it in the same terms. The thickness between the real base of the piece and the base where the fins start is different along the piece. In order to calculate the heat that would crosses the area considering the next graphic in the case of no fins, it is proposed the equation (4.15) beyond the graphic.

Figure 29. Thickness and areas



As is shown, the piece has been divided in seven portions with its different thickness ( $\delta_i$ ) and its different cross-sectional areas  $A_i$ . The boundary between areas is assumed to take place in the half thickness fin. The values of both parameters are calculated with the MS EXCEL cells and are shown in the table below. For every single case studied in this project are the same because of the unchanging geometry.

Table 10. Thickness and areas

$A_i \text{ (m}^2\text{)}$	$\delta_i \text{ (m)}$
0,008	0,006
0,0025	0,007
0,0025	0,0085
0,0025	0,00925
0,0025	0,00975
0,0025	0,01
0,005	0,01025

The expression that determines the heat transfer across these non-uniform areas is:

$$Q_{total, no \text{ fin}} = \sum_{i=1}^7 \dot{Q}_i = \sum_{i=1}^7 k \cdot A_i \cdot (T_b - T_{\infty}) \quad (4.15)$$

Where  $k$  is the conductance found in the next expression related with the heat transfer resistance:

$$\frac{1}{k} = \frac{1}{\alpha} + \frac{\bar{\delta}_i}{\lambda} \quad (4.16)$$

Eventually, the *overall effectiveness* is calculated as:

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,fin}}{\dot{Q}_{total,nofin}} \quad (4.17)$$

#### 4.2.4 Results and comparison

The next table shows the variables needed to calculate Fin Efficiency and Fin effectiveness with its values for every test made:

Table 11. Results and comparison

Material and Condition	$Q_{total} = Q_{total,fin} \text{ (W/m)}$	$Q_{max} \text{ (W/m)}$	$Q_{total,nofin} \text{ (W/m)}$
<b>Cooper <math>\alpha = 100 \text{ W/m}^2\text{K}</math></b>	1749,91	1962	101,77
<b>Cooper <math>\alpha = 6 \text{ W/m}^2\text{K}</math></b>	116,86	117,72	6,119
<b>Aluminum <math>\alpha = 100 \text{ W/m}^2\text{K}</math></b>	1587,42	1962	101,56
<b>Aluminum <math>\alpha = 6 \text{ W/m}^2\text{K}</math></b>	116,03	117,72	6,117
<b>Steel <math>\alpha = 100 \text{ W/m}^2\text{K}</math></b>	999,69	1962	100,07
<b>Steel <math>\alpha = 6 \text{ W/m}^2\text{K}</math></b>	110,60	117,72	6,113

Figure 30. Fin efficiency depending on the material and kind fo convection

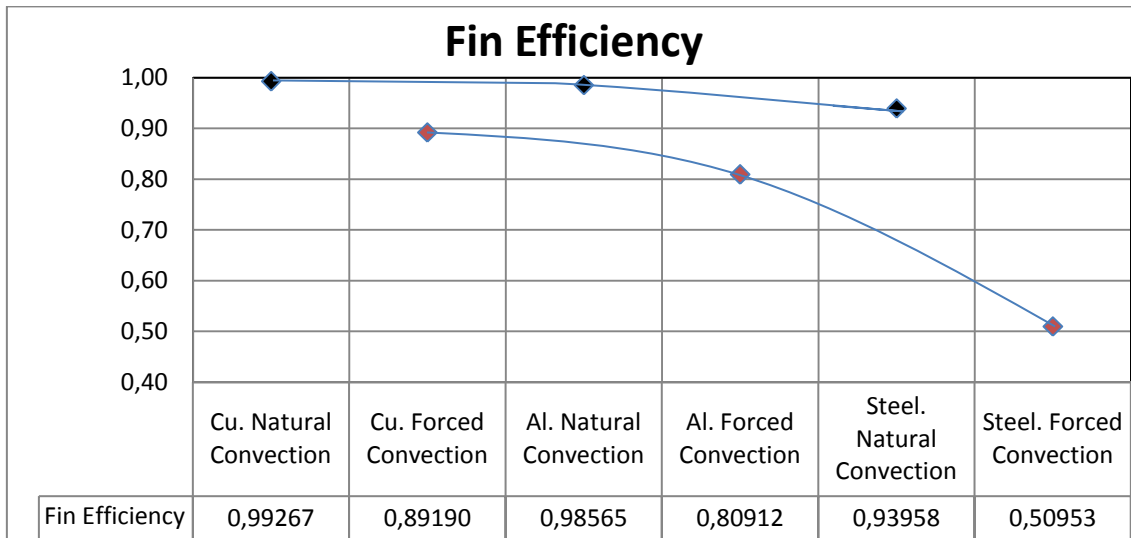
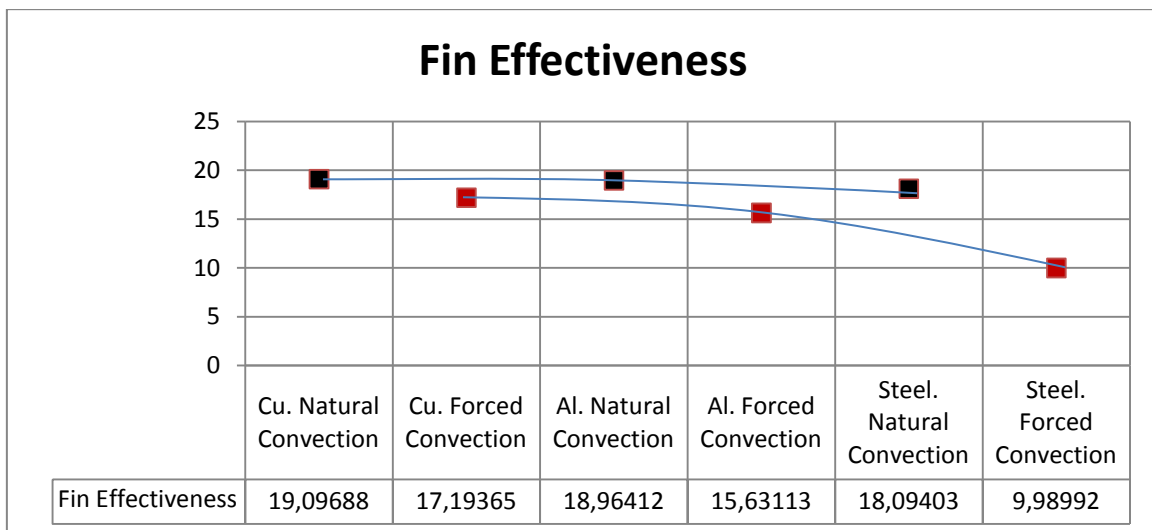


Figure 31. Fin effectiveness depending on the material and kind of convection:



As it was expected the material that presents more efficiency and effectiveness is the cooper due to a bigger conductivity ( $\lambda$ ).

In chapter 3 it has been demonstrated how the cooper was able to dissipate more heat under all kind of conditions. Remember that the efficiency is the comparison between the whole heat dissipated and the maximum heat that the piece could dissipate (only dependent of convection factor, geometry and difference of temperatures) is logical that cooper is the most efficient material. The maximum heat the piece could dissipate is equal (same conditions)

#### Reference used in chapter 4:

[6] Carslaw H.S. Jaeger J.C, "Conduction of heat in solids" 1959 Oxford

## 5 Analysis with ANSYS

The ANSYS program is capable of simulating problems in a wide range of engineering disciplines. In this project, it has been accomplished a *Thermal Analysis* focused in the first of the tests made with the objective of comparing results and methodology of work such as required time, automatic skills of the programs, etc.

The analysis with ANSYS addresses several different thermal problems, for instance:

- **Primary heat transfer:** Steady-state or transient conduction, convection and radiation
- **Phase Change:** Melting or freezing
- **Thermo mechanical Analysis:** Thermal analysis results are employed to compute displacement, stress, and strain fields due to differential thermal expansion.

ANSYS is a sort of program totally focused in the kind of studies carried out in this project. It offers a complete specific study in every field and its possibilities are wide. Lots of companies in the aeronautical field trust in this program to make the required tests. It is necessary to highlight the importance of the experience developed by the ANSYS after several years solving problems similar to this study.

In the next chapters is possible to find a brief description of the methodology used to accomplish the study of one of the models already analyzed in MS EXCEL. In these paragraphs are shown the necessary steps the user should follow to achieve a complete study of the piece as well as the results provided by the program.

We may notice that the timing and manual work required by ANSYS is minimum and the power of the program to achieve automatic calculations is huge.

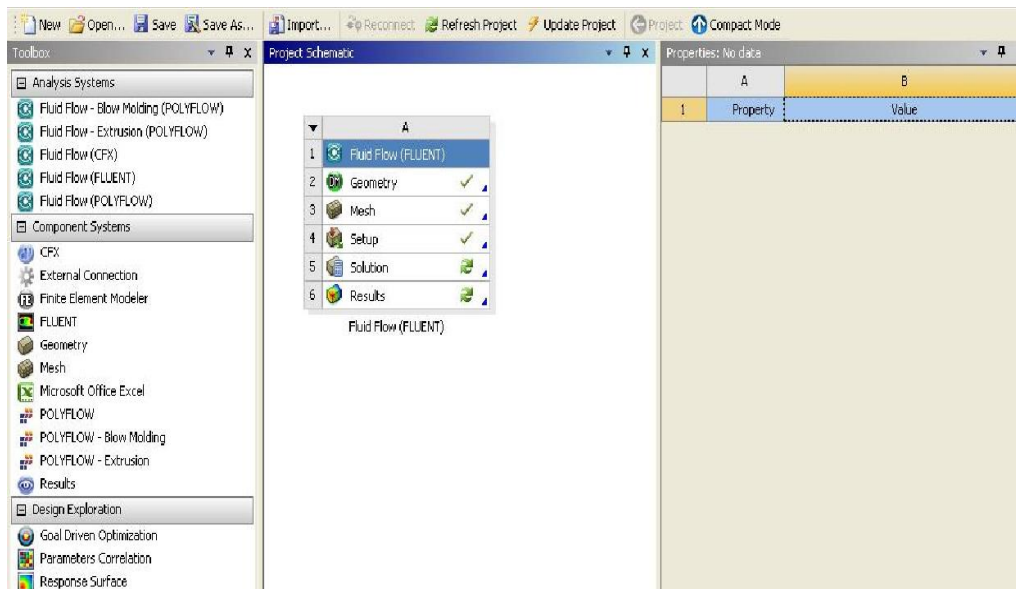
### 5.1 Description of the method

The first requirement needed by ANSYS to start the modeling of any piece in the thermal problems is creating single elements such as the geometry or the mesh. It means describe the piece of the study.



Once is set the option FluidFlux (FLUENT), is possible to select the options shown in the image below in order to describe geometry, mesh, etc. This first square summarizes the whole analysis and the specifications will be set every time we select one of its options.

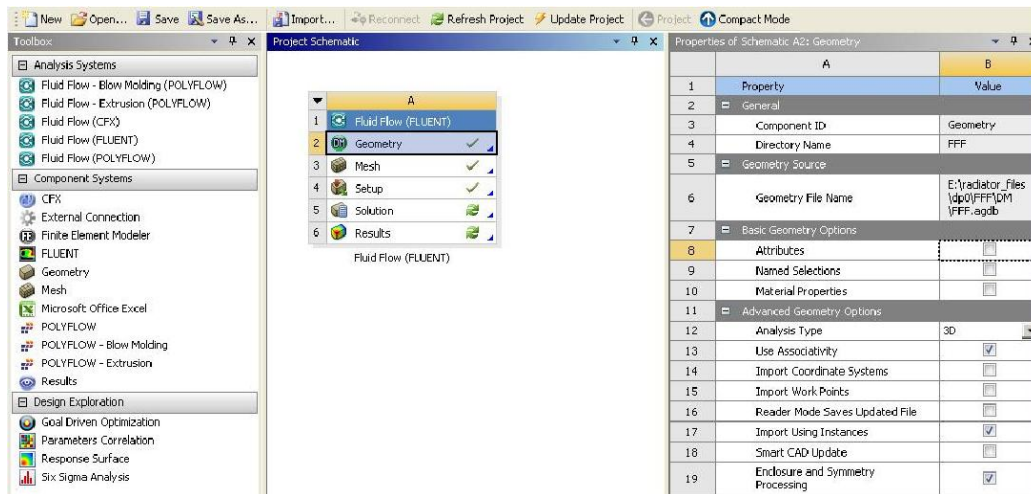
Figure 32. Workbench



### 5.1.1 Geometry and meshing

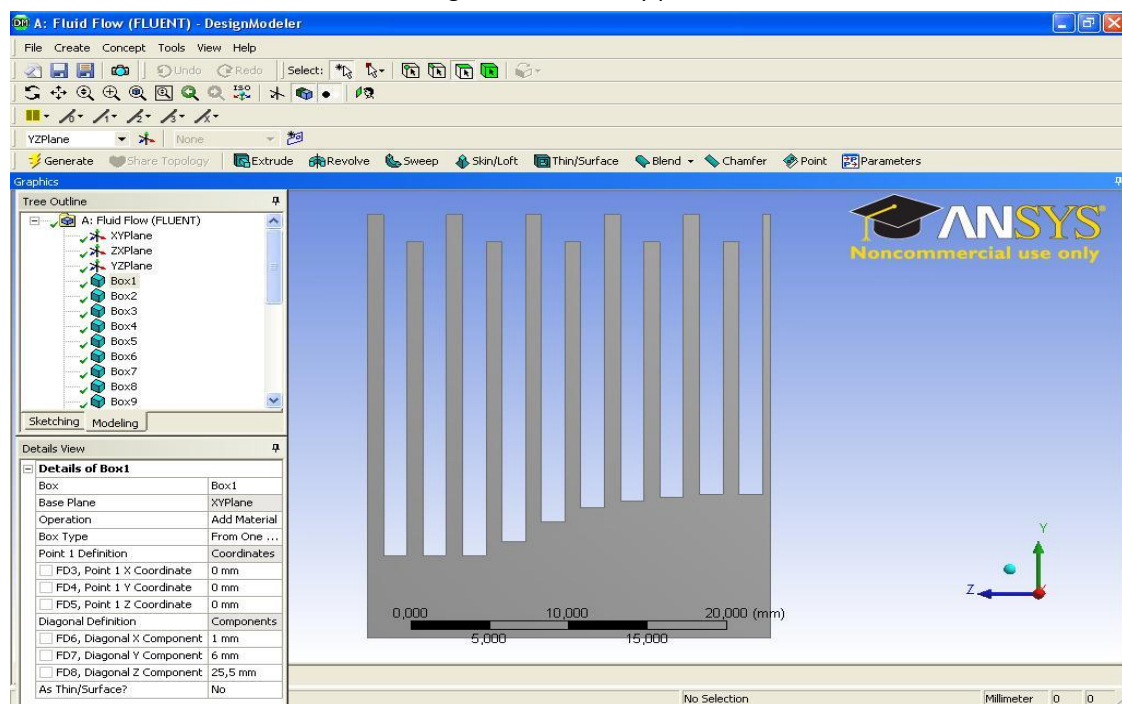
If we start with the geometry selecting *geometry* several options are added to concrete the geometry properties. In the next graphic we can notice the different options that can provide the program before start drawing the geometry of the body. The geometry can either be drawn from the beginning using the work sheet provided by ANSYS or importing it from other programs such as SOLIDWORKS or SOLIDEDGE.

Figure 33. Geometry



If we proceed drawing the piece by selection of *NewGeometry* the program provides a work sheet very similar to any drawing program. In the case of our piece, the accomplishment of our geometry is done with the setting of several *box*, an option given by the program. Then, is defined the origin of coordinates, the length, the width and the depth of it. The assembly of all the boxes done complete the geometry of the piece, the methodology fits perfectly our case due to the squared-shape of the body. The results as well as the option offered, in addition to the *boxes* used are shown in the next image.

Figure 34. Geometry performance



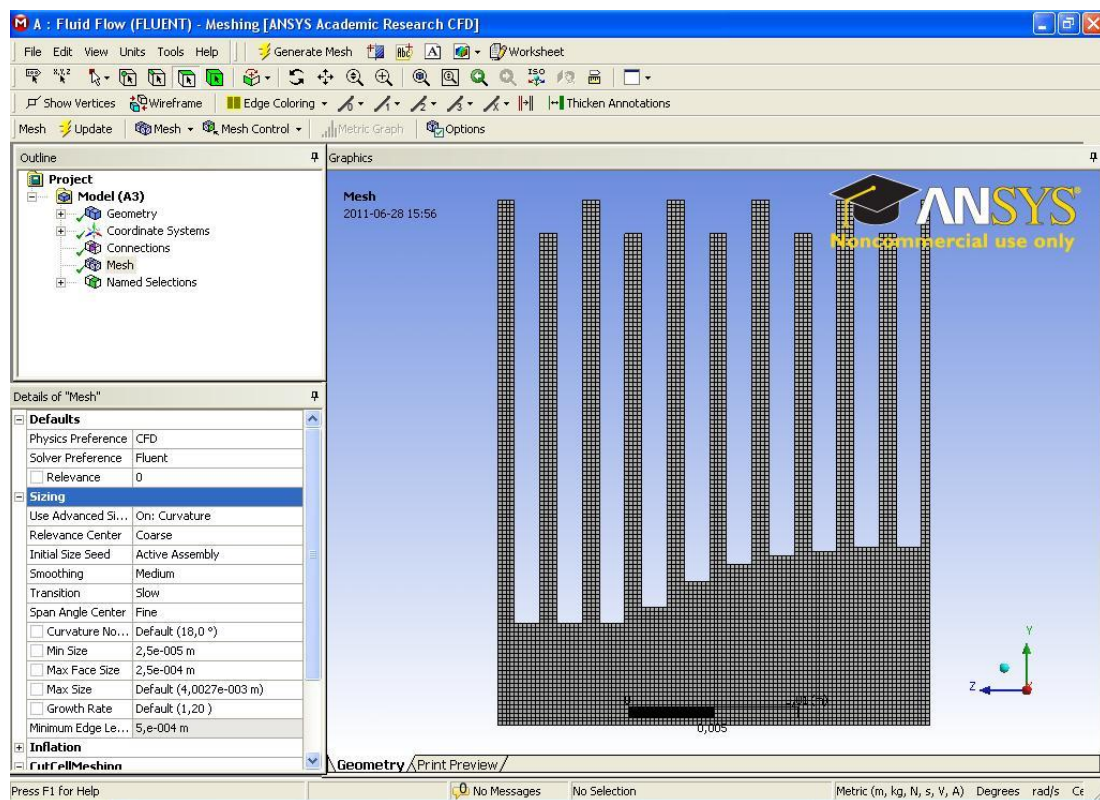
Once the geometry is performed, the next step is doing the meshing of the piece. The meshing consists in the discretization of the body through little cells. In order to fit the MS EXCEL performance, the dimensions of the little cells in which we divide our piece are equal:

$$\Delta x = 0,25 \text{ mm}$$

$$\Delta y = 0,25 \text{ mm}$$

In the main screen we select *mesh* and several options are presented referred to the geometry already created, from these options is possible to specify the commented dimensions of the meshing on the folder *seizing* showed in the next ANSYS screen:

Figure 35.Meshing



The advantage of the *meshing* in ANSYS is clear, everything is automatic after set the details, in our project not many specifications are required:

$$\text{Min. Size} = 0,25 \text{ mm}$$

$$\text{Max. Size} = 0,25 \text{ mm}$$

In the same folder as *Mesh* it is possible to define the boundary conditions clicking in every surface of the body in contact with the environment.

We have defined both needed boundary condition for this specific case following these next two steps:

*Bottom/Secondary Button of mouse/NamedSelection\_Boundary1/Apply/Generate*

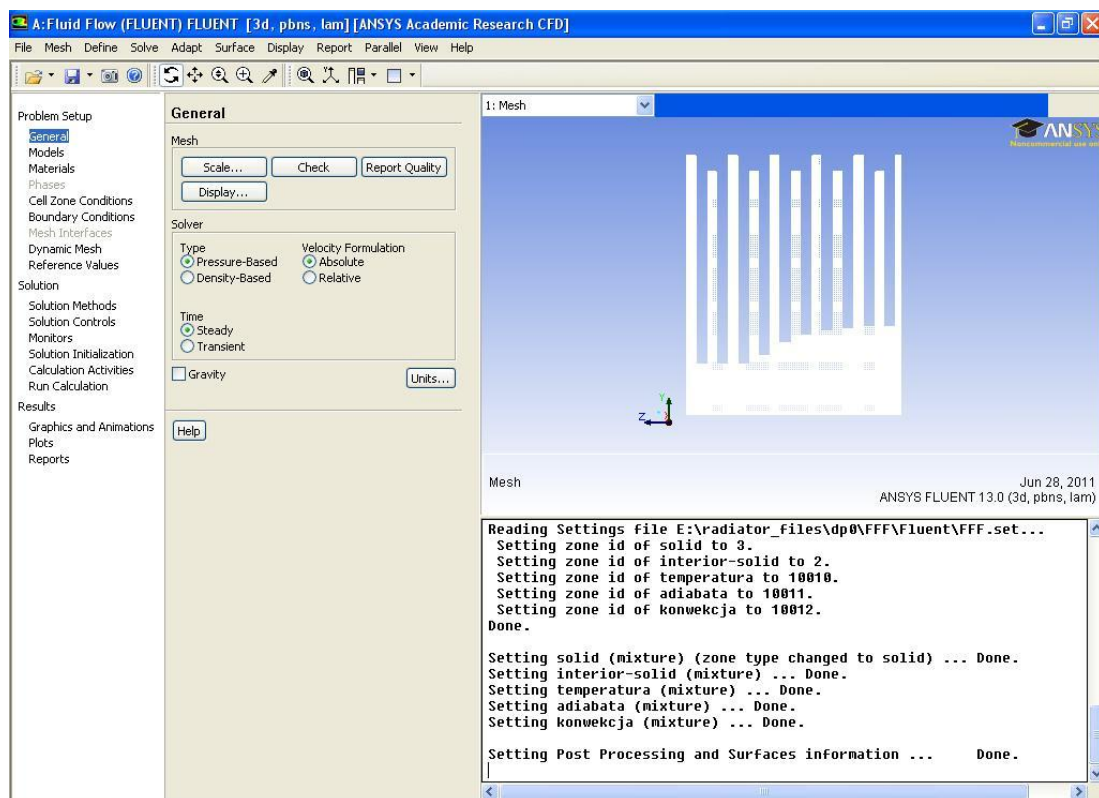
*Surface by convection/Secondary Button/NamedSelection\_Boundary3/Apply/Generate*

### 5.1.2 Setup

After define the boundary conditions in the surface we select *setup* in the main screen. In the *setup* folder we will find the options we should set to solve the problem. Are divided in:

**Problem Setup:** Description of the problem by general considerations such as steady or time-dependent, materials used with the proper values of conductivity and boundary condition.

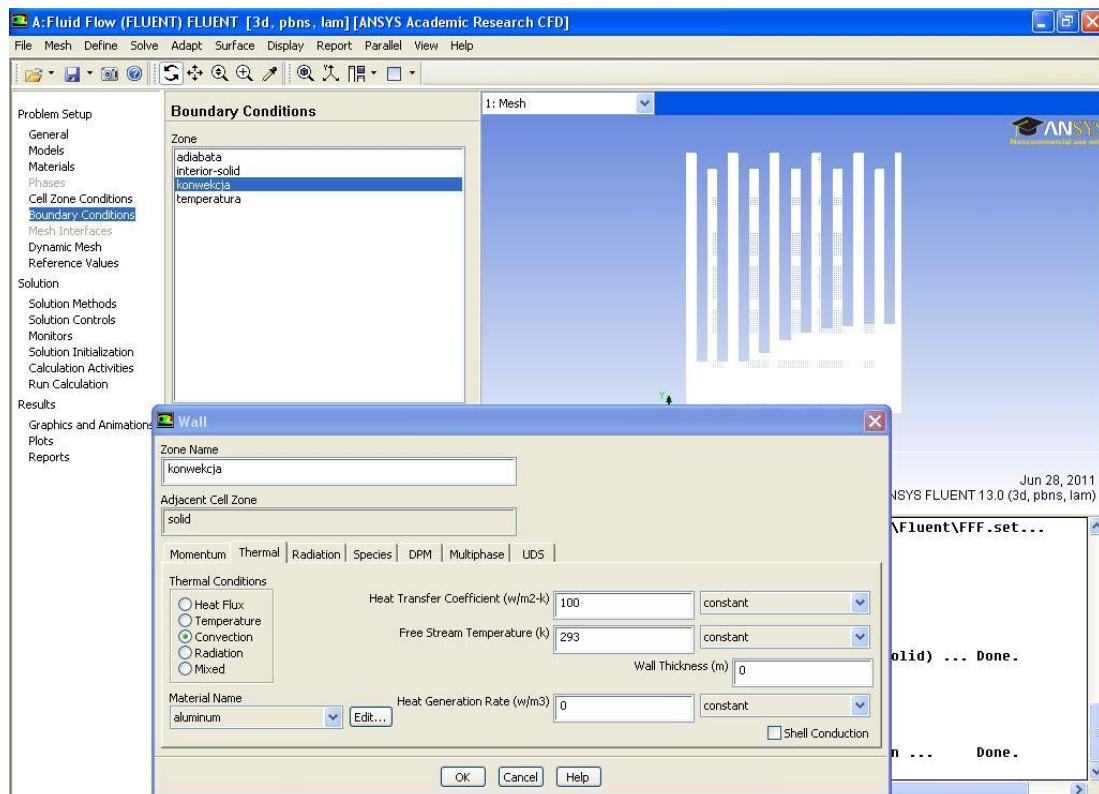
Figure 36. Setup – general options



It is also necessary to set the *Model* folder (*Energy ON*) and specify the *Materials* (*Fluid: Air & Solid: Al*, in our case),

One of the main steps is the right choice and setting of the *boundary condition*. In the next graphic it is shown the needed specifications; notice that all the factors, temperatures and type of heat flux are set in the window named *Wall*. Thus, all the values are ready from this moment to the next determinations.

Figure 37. Boundary conditions



**Solutions:** After the selection of *Initialize* in the *Solution Initialization* folder to prepare the calculations, the main factor to describe in this paragraph is the *Run Calculation* which provides the wanted iterations in order to solve the problem. As MS EXCEL, ANSYS use iterative calculations to achieve heat and temperature distribution.

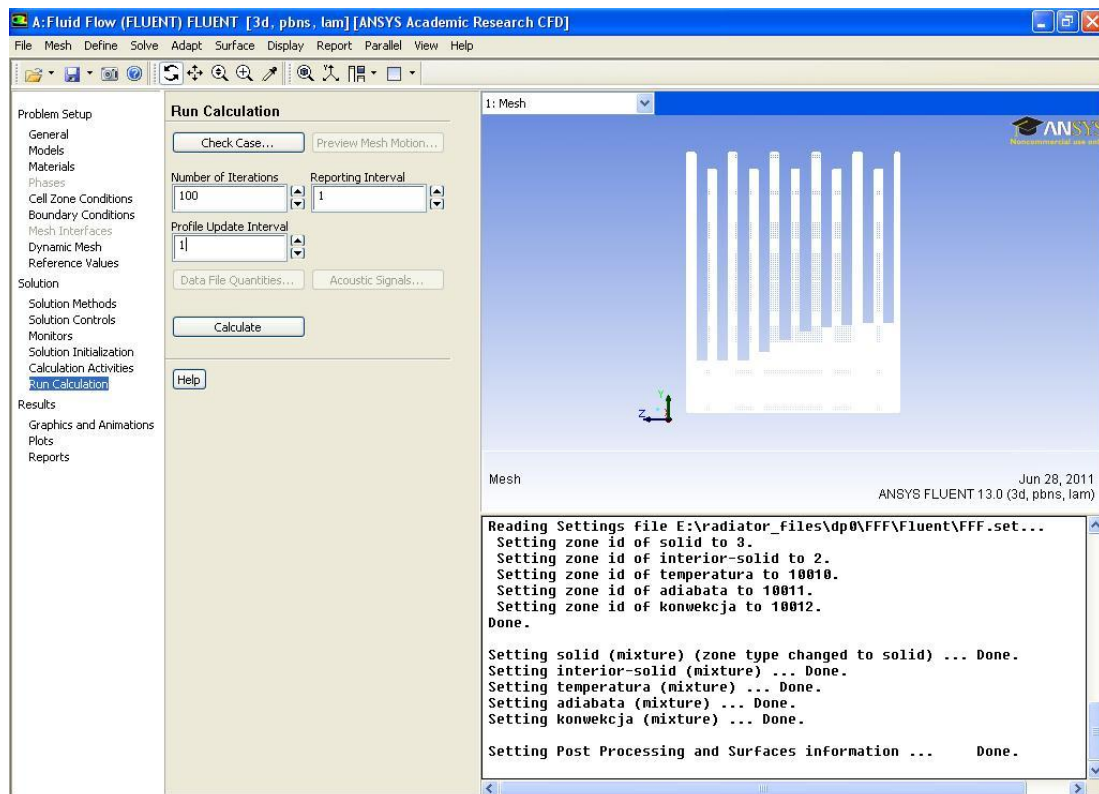
The values that we should specify after preparing the solution of the problem are:

- Number of Iterations
- Reporting Interval
- Profile Update Interval



In the picture above are shown the settings for start solving the problem. It may notice that the number of iterations to begin is 100, an arbitrary value. Afterwards, the program solves the temperature and heat distribution with the iterations that it really needs, but never more that the value indicated.

Figure 38. Run calculation



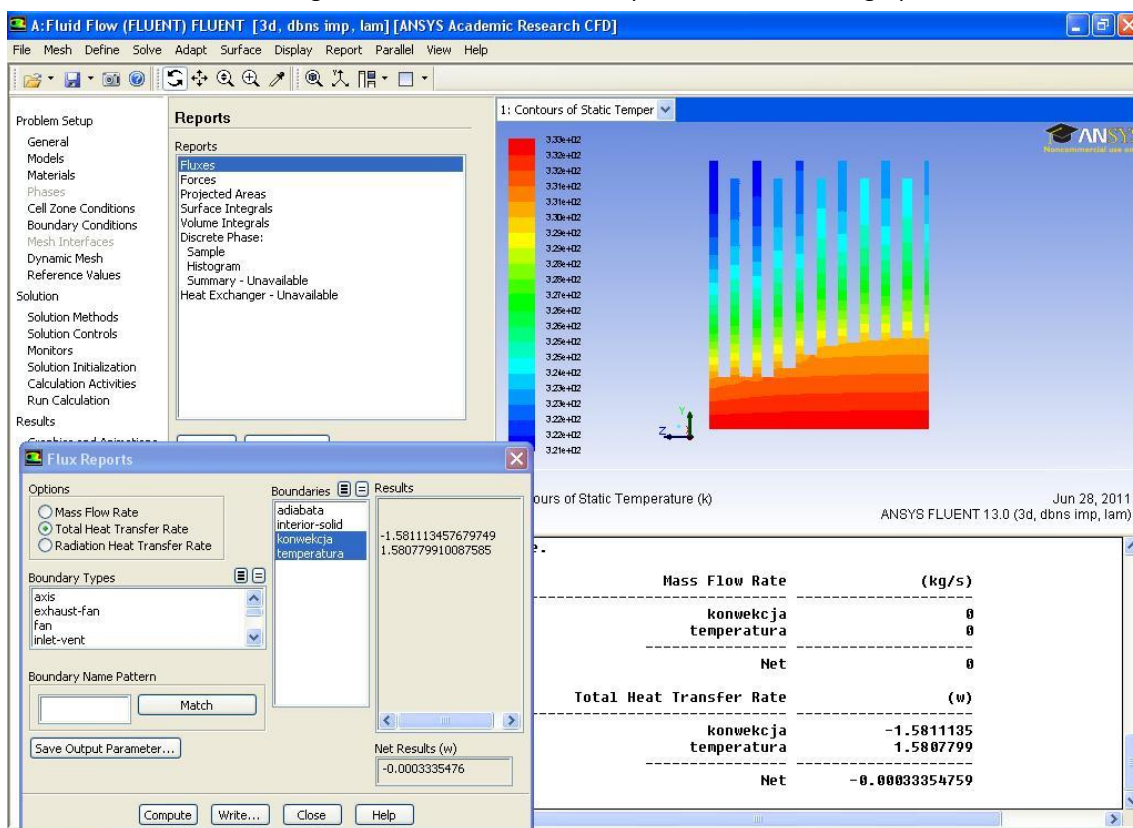
The program does not need a lot of iterations because of the steady conditions set in previous options.

**Results:** Eventually is the last screen that will appear and it shows the required solution. In the next paragraphs is shown the results founds as well as the graphics with the temperature distribution. May be noticed that the solution found has been developed automatically for the program once has been indicated the iterative calculations. The folder *Graphic and Animation* shows the temperature distribution and the folder *Reports* provide the values of heat transfer.

## 5.2 Results

In the next image is possible to have a look of the results given by ANSYS. The performance has been done always with 1 mm depth whereas in MS EXCEL is considered as 1 meter = 1000 mm. This is the reason why the results are 1000 times lower. The solution is given for the heat flux dissipate by convection (konwekcja) and for the heat flux that crosses the base, where the temperature is prescribed (temperatura).

Figure 39. Heat results and temperature distribution graphic



Heat dissipated by convection: **1,5811 W/mm**

Heat crossing the base piece: **1,5807 W/mm**

## 6 Conclusions

---

### 6.1 Comparison between ANSYS and MS EXCEL

As it has been remarked along the project, the comparison between these two programs does not occur in the same level when the aim is solve a heat transfer problem. ANSYS is specialized in this sort of procedures whereas MS EXCEL consists in a huge calculation sheet which possibilities are wide, thing that has allowed us to perform a heat transfer problem and find not only a solution but also a methodology to comprehend and develop Numerical Method problems.

ANSYS is focused in provide a solution for any heat transfer problem, and the results as well as the methodology is accurate and built easily. On the other hand, almost every step followed in MS EXCEL requires manual settings and a basic knowledge in the subject. Therefore, ANSYS shows the values of the solutions fluently, without taking care of formulas or previous steps in which calculations are required, whereas MS EXCEL helps to develop both preparation and solution of the problems.

One of the main differences of these programs is the **time** needed to describe and solve the problems. As it has been shown in chapter 5, ANSYS program does not need a lot of specifications in all the areas, just a description of the problem wanted to solve using the wide range of options the program offers. Almost every step the user wishes to specify is able to be developed by one of the ANSYS options such as the kind of heat flux: convection, conduction, etc. Thus, the time the user spends in every step is insignificant compared with the MS EXCEL. No formulas have to be settled because the program already has it. However, in MS EXCEL there is no option in which the user might choose *convection* or *conduction*, and the way to be able to set any condition is by the writing of the formulas belonging to each case. The process is longer but, as it has been demonstrated, once a first test is made, the others can be built from previous descriptions by changing some parameters. This is one of the advantages the MS EXCEL offers, the possibility of moving and copying formulas, spreadsheets or even entire documents.

Other example is the **geometry**, in this case the difference is not as important in required time as it is in the way the program allows the user to draw any geometry. In ANSYS the geometry is accurately drawn with a methodology very similar to the one used in programs



such as SolidWorks (program specialized in technical drawing). Consequently, the way the user builds up the geometry is supposed to be fast and easy but it requires quite experience in drawings programs. In this case, the possibility to draw simple shapes in MS EXCEL is very simple and any user experienced in computers (nowadays every engineer) is able to prepare geometry without previous lessons. Also, the time in which the geometry was performance in this project was less in MS EXCEL than in ANSYS.

One of the main differences lies in the **manual** requirements of each program. It is known that ANSYS applies automatically most of the steps needed to create and solve a problem, but MS EXCEL needs manual process during the problem description. The obvious inconvenient is the time needed to specify all that options that ANSYS performs by itself. However, along the accomplishment of the test made in MS EXCEL, the user actually realizes and understands the steps that he/she is carrying out. In other words, MS EXCEL allows the user to get into each case and comprehend it, for instance, by the settings of formulas. In this way, MS EXCEL is used as a learning tool to understand what Numerical Methods really is and a practical method to find results and check the coherency of them. On the other hand, there are cases such as editing the temperature distribution graphic format that requires a lot of time in order to arrange the colors, for example.

Along all the work develop in this project and by the usage of the predicting method described in chapter 2, it has been demonstrated that the **number of iterations** needed to find coherent solutions with MS EXCEL are less than the **iterations** the program would need to converge. No comparison with ANSYS is possible because the power of the *fluent program* to determine the final solution with both heat transfer rates and temperature graphic is huge. Whereas the ANSYS has needed around 10 iterations to find out the solutions of the problem, in MS EXCEL for the same case were necessary around 60000 to arrive to a result with 0,0001% error. However, the possibility of using macros (chapter 2.4.1) in MS EXCEL provides a comfortable way to accomplish these iterations.

The possibility that MS EXCEL, as a calculation sheet program, offers to present **results** in tabs shapes and graphics has been important to complete this project. Once the iterations were made in each case, the results could be shown and used in other spreadsheet in order to calculate other interesting factors beyond the heat transfer. Therefore, MS EXCEL has not only helped with the running calculations explained in this chapter but also to achieve *efficiency*

and effectiveness of the fins piece, comparison graphics to present the different values attained with *natural or forced convection* in the environment...

Eventually, one of the most important targets of this thesis has been the demonstration of solving a heat transfer problem with the usage of a relative cheap program, specially compared with ANSYS. The **cost difference** between the programs is huge and most of the universities need a tool to explain Numerical Method and show how the solutions of such problems are carried out technologically. Nowadays ANSYS license is unaffordable for a big part of Europe universities and the possibility to create an alternative method was the main aim for the MS EXCEL use.

## 6.2 Effectiveness and Accuracy in MS EXCEL

Despite of the time needed to accomplish this project and taking into account that the writer had not had any knowledge of the method before starting, the results found after the 5 month experience are satisfactory, and every test made has finished successfully. In every single case studied we have achieved a converging solution and a number of iterations where the heat value predicted was approached to the converging one.

Bearing in mind all the differences explained in the previous paragraphs with a reference program in the heat transfer field, the usage of MS EXCEL to explain, learn and teach numerical method problems for simple piece shapes is completely effective.

The accuracy of the method is shown in all test already made and explained in these pages. If we take as a reference the first one made (Aluminum, forced convection and prescribed temperature in the base) the heat transferred is totally converged after 158000 iterations and both  $Q_w$  and  $Q_d$  reach 1587,486195 W/m, that means that they are equal until the sixth decimal. It is important to remark that the body shape analyzed in this project consists in a finned surface, fact that slows down all calculations compared with a regular shape because of its complexity. In ANSYS the values obtained are  $Q_w = 1581,1 \text{ W/m}$  and  $Q_d = 1580,7 \text{ W/m}$  after a reduced number of iterations.

Eventually, as an ending conclusion, MS EXCEL provides an interesting and didactic way to achieve enough accurate results in the study of heat transfer by Numerical Method. The possibility of develop both announce and solution of the problems offers a general view of the

meaning of Numerical Method and the reliability of its solution allows its usage in practical cases as the piece study in this project.